Dear Colleagues,

I am experimenting with OpenOffice.org as a quick means to organize some drawings and text. In the past, I have used a desktop publishing program but I am hoping this will be easier.

Now, here is the real Самиздат:

Normal and Anomalous Color Vision

I started talking to a geneticist in a coffee shop and the conversation naturally turned to genetics of color vision. I have had the vague idea that vectorial color might have application in describing anomalous versus normal cone responses. A quick search uncovered an article with not only graphs, but also tabulated sensitivities for normal cones, and for anomalous red and green cones: Paul DeMarco Joel Pokorny, and Vivianne C. Smith, "Full-spectrum cone sensitivity functions for X-chromosome-linked anomalous trichromats," *J. Opt. Soc, Am* A **9**(9):1465-1476.
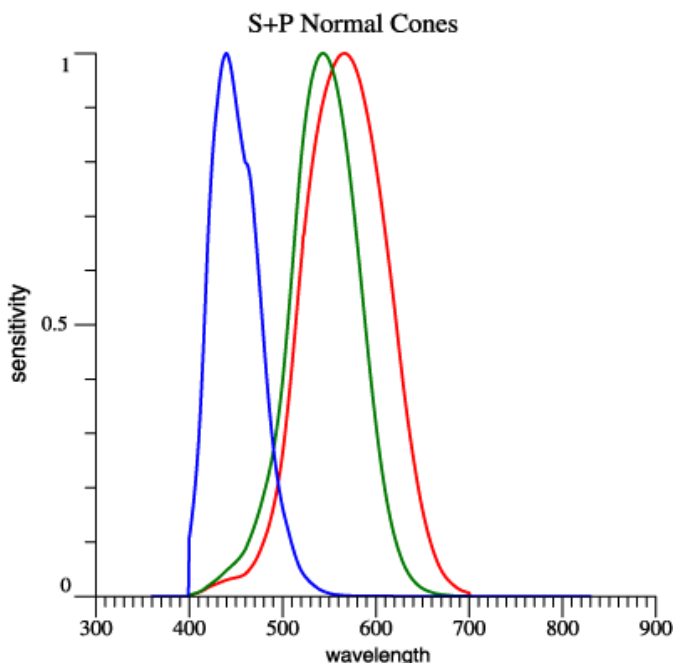
The functions above and on pp. 2-3 are from DeMarco's article. If you would like these functions as files, please contact me, as it was some trouble to extract them from the article, even though I was copying and pasting from a pdf file. Also, if you would like the pdf of the DeMarco article, I can send it. Beyond the S+P cone functions themselves, I extracted 2 more columns of numbers, for the anomalous red and anomalous green cones. There are some additional columns that I did not extract.

All 5 Cone Types

*(vertical axis: sensitivity, from 0 to 1; horizontal axis: wavelength, from 300 to 900)*
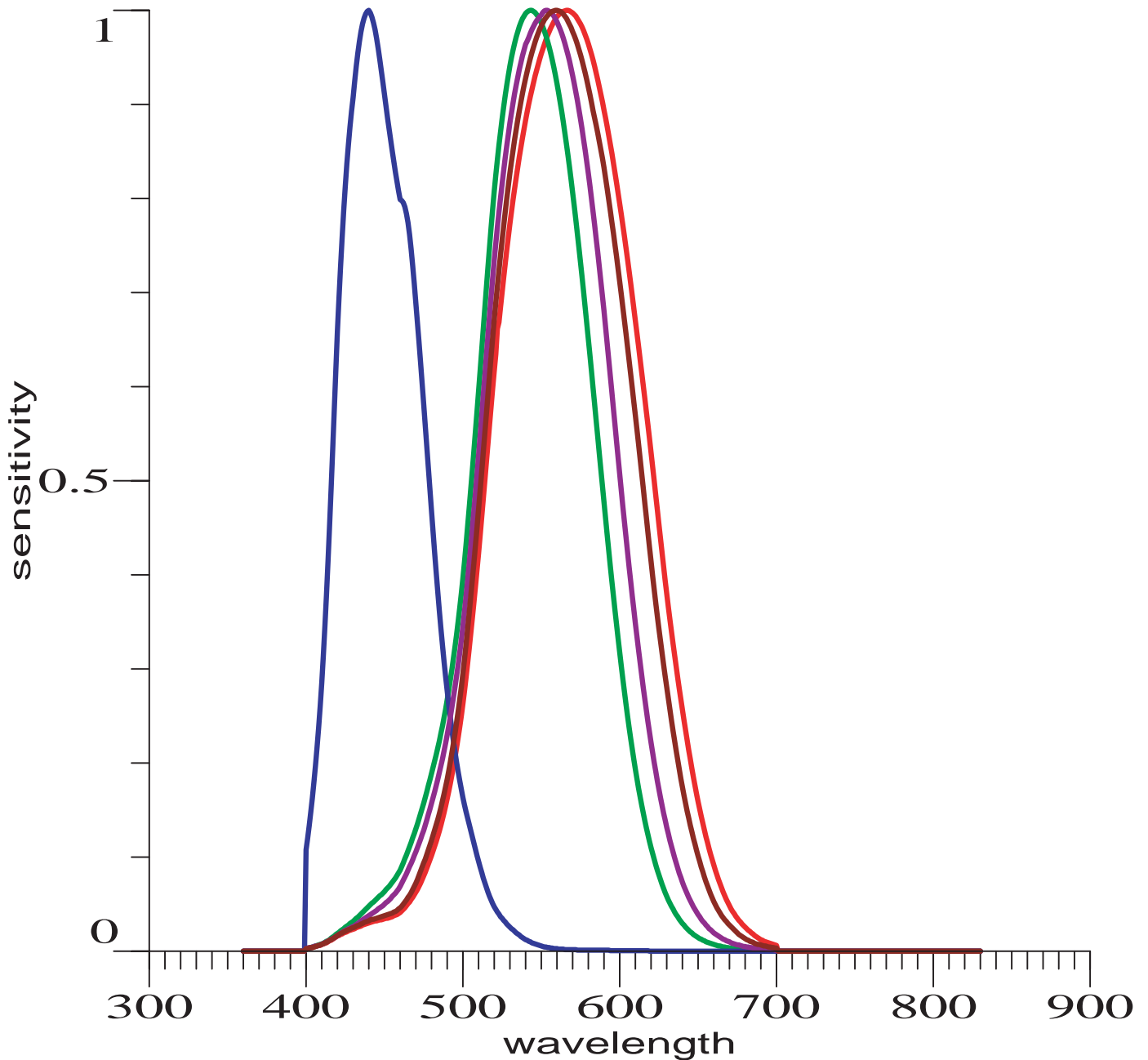
red, green, blue = Smith + Pokorny normal cones
purple = anomalous red cones
maroon = anomalous green cones
Yes, anomalous green is at longer λ than anomalous red

These next graphics are self-explanatory. Various pairings can be of interest. You want to know how much the anomalous red pigment differs from normal red, but in life the red-anomalous person will have the anomalous red pigment in combination with normal green. (Presumably, as far as I know.) Thus, the pairings of anomalous red with normal green, and normal red with anomalous green, are the working combinations.

S+P & Anomalous Red Cones

red = normal red cones
purple = anomalous red cones

S+P and Anomalous Green Cones

green = normal green cones
maroon = anomalous green cones

S+P Red & Anomalous Green Cones

red = normal red cones
maroon = anomalous green cones

S+P Green & Anomalous Red Cones

green = normal green cones
purple = anomalous red cones
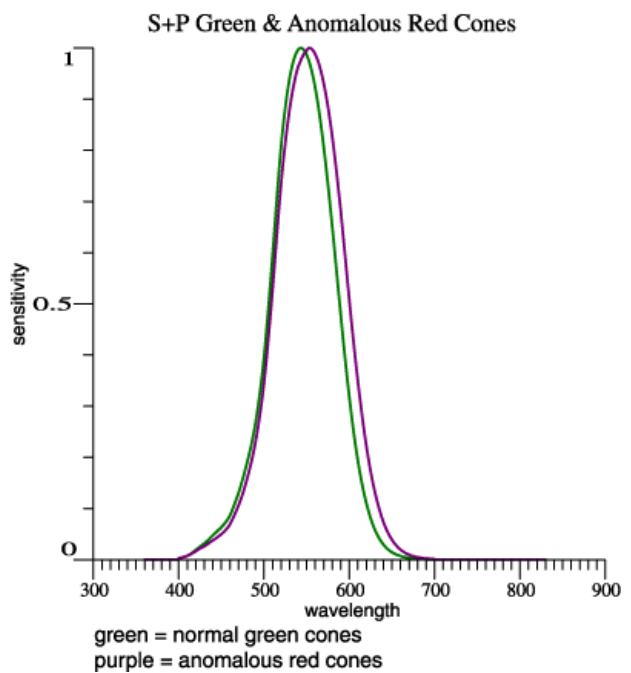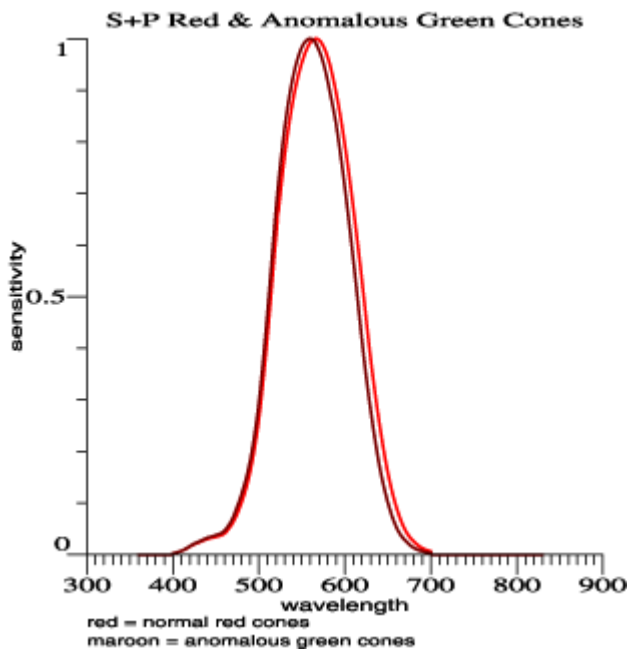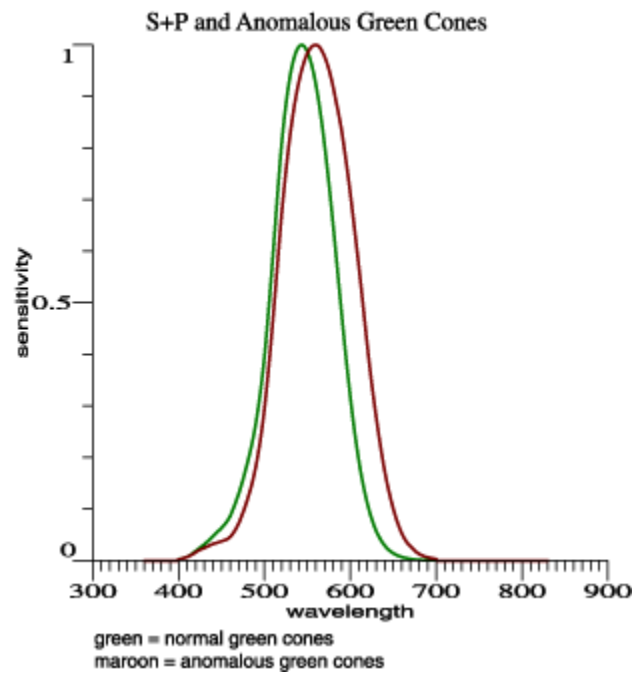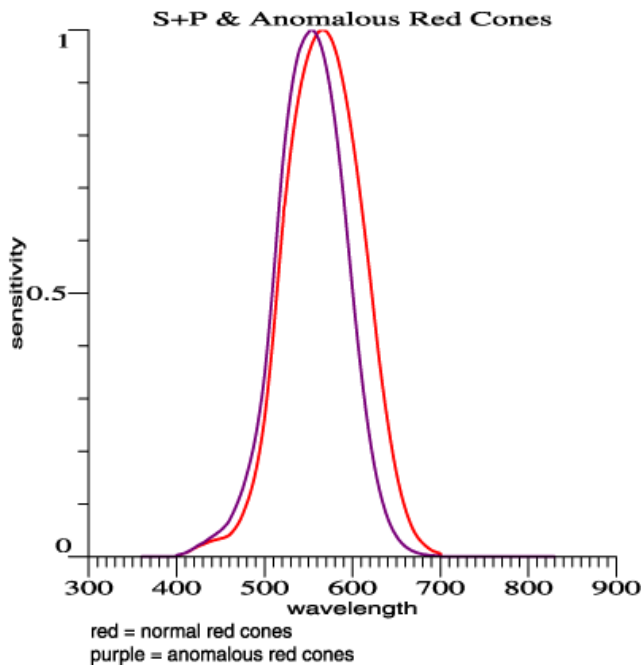
So far, these are preliminaries. Now suppose we take a normal person's cones, orthonormalize the functions and make a parametric plot. That would be the Locus of Unit Monochromats, except for any discrepancybetween the 2° observer and the Smith & Pokorny cones. Then if we calculate a Locus of Unit Monochromats for an anomalous observer, would it differ in a revealing and helpful way? Any insight that we gain may also apply to such topics as variations in normal color vision, or camera sensitivity functions.

Because 3D plots are hard to work with, parametric plots will be in 2 dimensions. Let's see what happens.
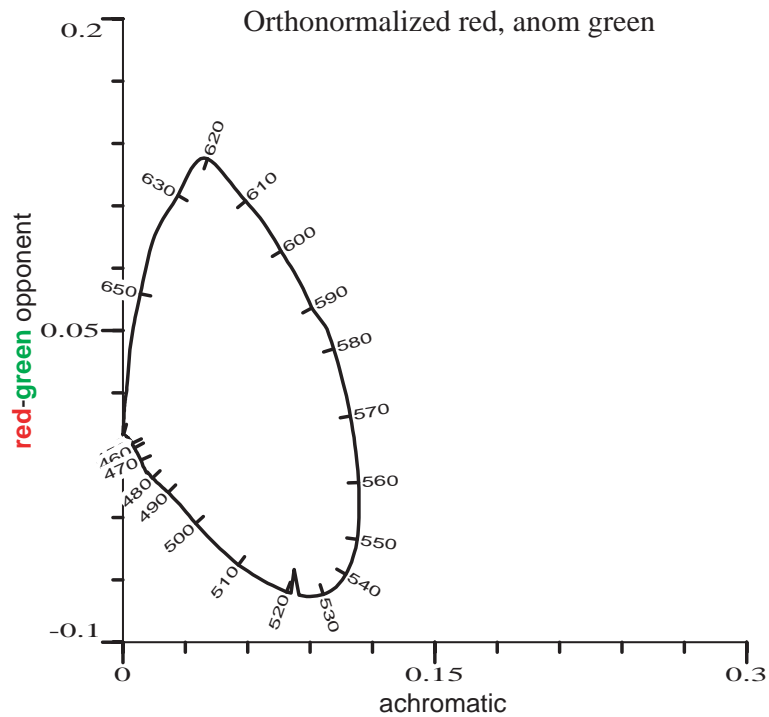
Here is a 2D locus of unit monochromats for red and green cones:

Orthonormalized S+P r-g
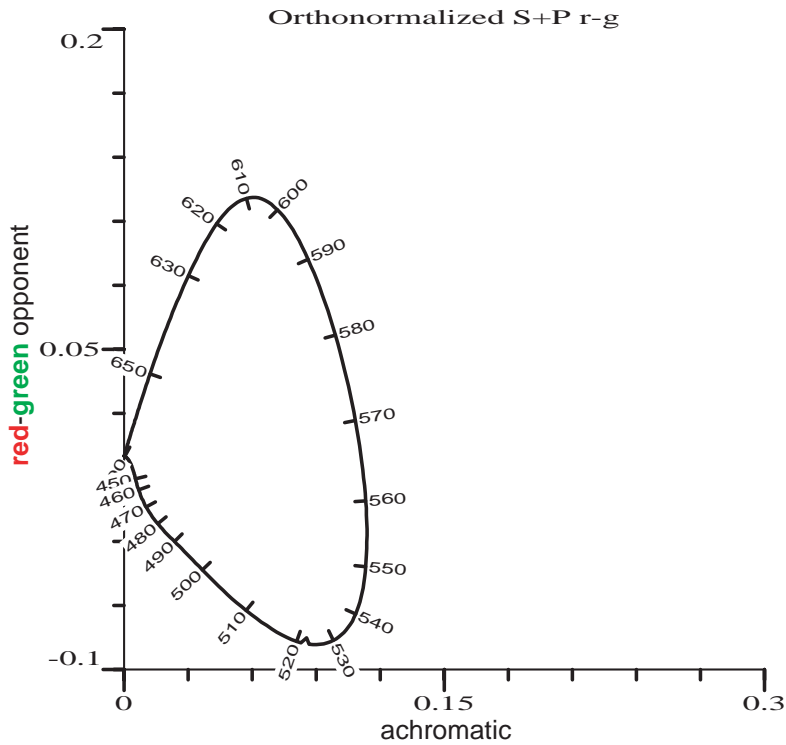
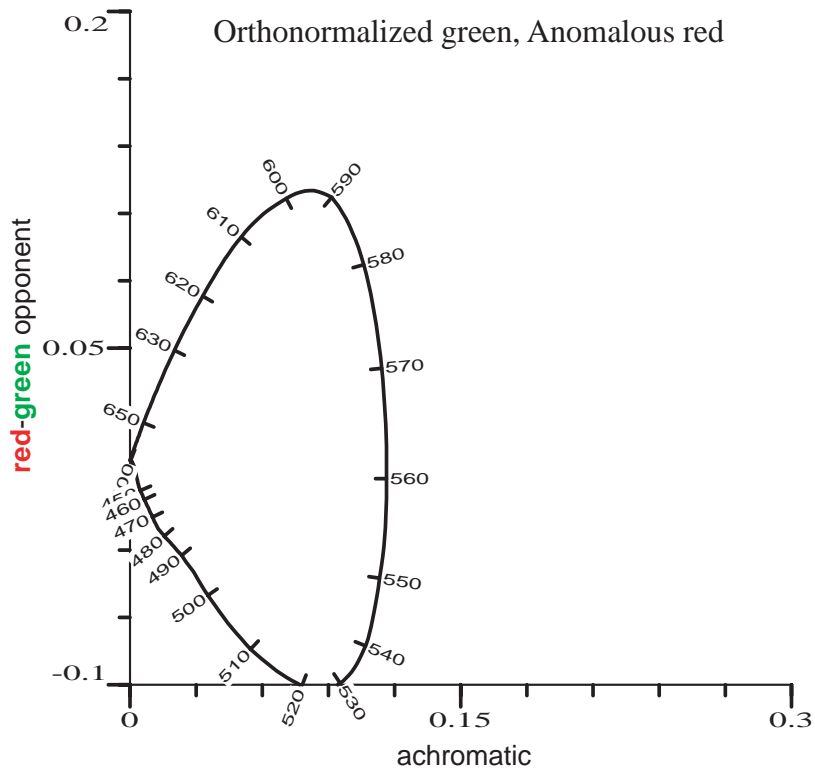Here is a similar locus when anomalous green pigment is substituted for normal green pigment:

Orthonormalized red, anom green

They're a little different, but it's hard to name the striking distinction. We may as well look at normal green with anomalous red:

Orthonormalized green, Anomalous red

Again, there may be some interesting bit of information. If we want to identify a critical difference between the normal and the anomalous observer, or between the 2 anomalous types, these graphics could be a starting point.

How did I choose the achromatic dimension, starting only with 2 cone sensitivities? It's pretty darned simple. I make a 2-column matrix such as $\mathbf{A}$ = [S+P red, Anomalous green]. Then compute "matrix $\mathbf{R}$:"

$$\mathbf{R} = \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T \ ,$$

as every schoolgirl knows. Then take old-fashioned y-bar for the 2° observer and compute the matrix product $\mathbf{R}*$(y-bar), that's the achromatic function for this green-anomalous observer. Of course, Smith and Pokorny would sniff at this, they can find the subject's y-bar by flicker photometry, for example. They are right, but I don't mean the directions to have much significance. Anyway, even Smith and Pokorny might want to express their y-bar as a linear combination of red and green cones. My goal is to focus on the neglected analysis of the transduction stage itself. The shape of the locus is invariant; we should try to make inferences from the locus shape, not its exact orientation.

So, the most obvious issue might be the loss of signal strength in the opponent channel for the anomalous observers, but that is **not** captured in the locus of unit monochromats. It is undone by the normalization step in the Gram-Schmidt process. The loss of signal is better portrayed in graphs that I think of as Cornsweet plots. In Cornsweet's book, he doesn't mention Gram-Schmidt, but he plots one cone stimulus against another. This next graph is a Cornsweet-type plot for the normal S+P cones.

## Red stim vs Green stim, S+P cones

both normal cones
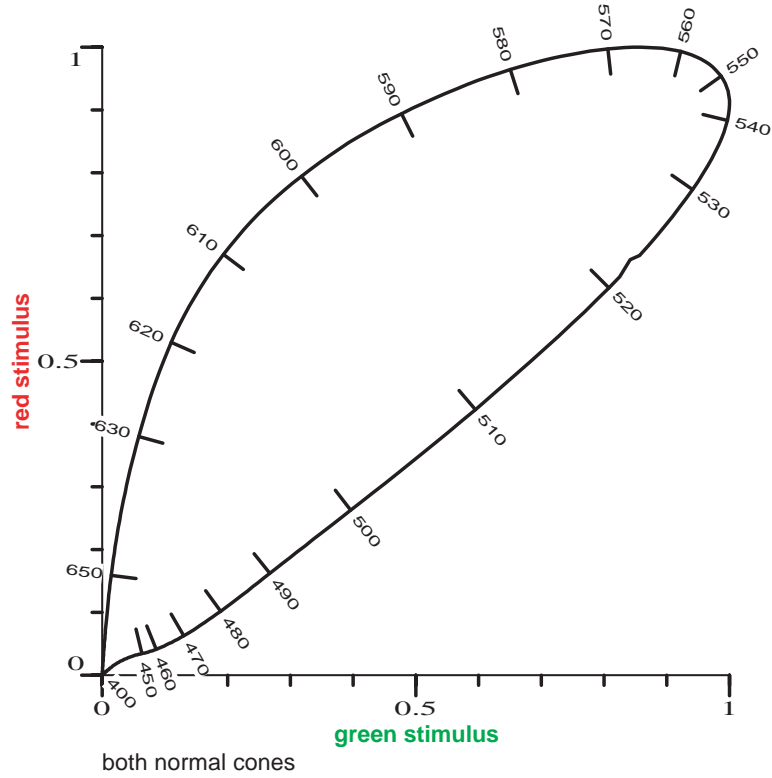
Next is anomalous red stimulus versus the same normal green. Now the smaller area makes it clear that the 2 cone stimuli are more correlated. The opponent signal must be weaker. But what does that mean?

Thu 2005 Mar 10, 20:12:07

Anomalous Red stim vs Green stim
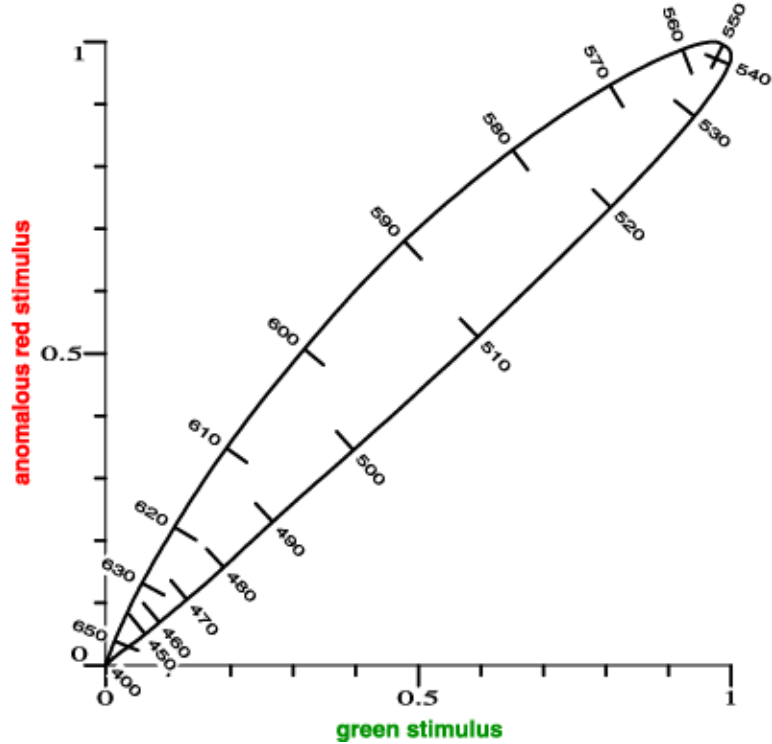
Here is the Cornsweet-type plot for the green anomalous observer. Lacking further information about cone populations or whatever, we can say that the green anomalous fellow has worse signal-to-noise than

Red stimulus vs Anomalous Green stimulus
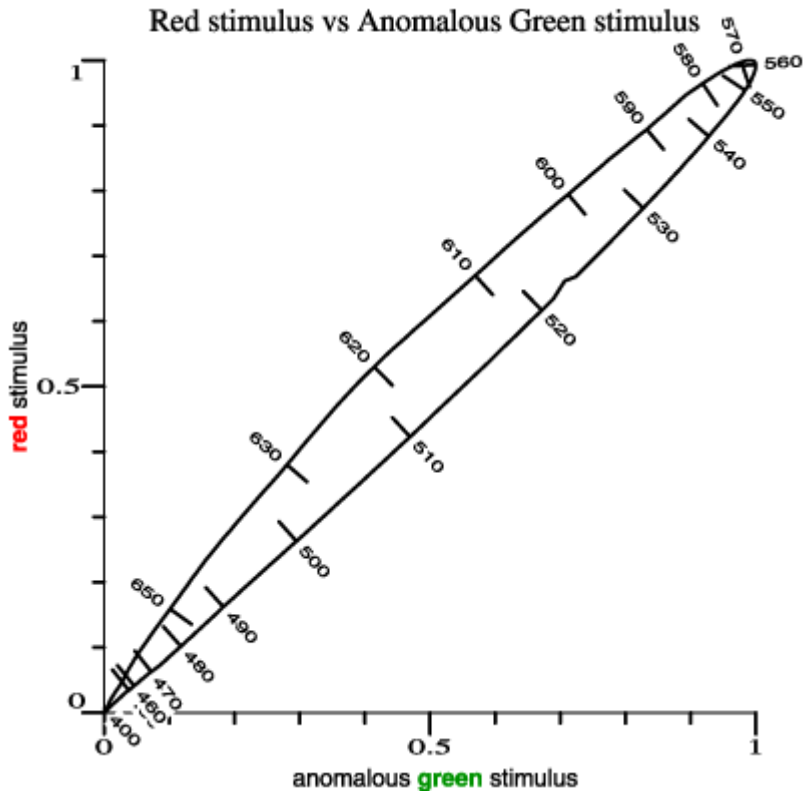
the red anomalous person. That is, the area is smaller. DeMarco et al. adjust each cone sensitivity to peak at 1.0. Therefore in these last 3 plots, I have not calculated anything, I just imitated Cornsweet by plotting the one function against the other.

Question: Is there a simple way to express the signal/noise issues for these systems? It seems to me that there would be a dark current and then shot noise for each receptor would relate to the quantum catch. For a "unit monochromat," a narrow-band light at unit power, the quantum catch would be proportional to sensitivity/wavelength. Etc, etc, but I don't what's been said before or who said it. What do you think?

What I'm thinking is, if the orthonormalized basis functions lose the signal-to-noise information then it would be appropriate to explain or cite something about signal-to-noise as a separate issue. The orthonormal basis (or matrix **R**) embodies the *shapes* of the cone sensitivities and nothing else. A person could say this and then add emphasis by referring to the Cornsweet plots and/or simple signal/noise formulas. Guth gave his opponent functions in such a way that blue has low amplitude, something that always bothered me because the shape of the functions may be more important than the amplitude issue that he had in mind when he did that. I'm not criticizing Guth, but I'm saying that this is one of those issues that remained murky in the 20th century. Jozef Cohen showed the importance of shape in itself, then amplitude is a separate issue. Color anomalous observers stimulate us to think about signal/noise.

The next graphic gets at the issue of prime colors. The abscissa is labeled Distim Radius, by which I mean radius in orthonormalized distimulus space, since only red and green cones are being considered. This value is also the square root of the diagonal of Matrix R, since that is one way to calculate that radius. After the graphic are some raw results from my program. I asked it to look for the local minima and maxima. You can see here why I like very smooth "cones" computed from the 2° observer. They give fewer false mins and maxs under a simple search algorithm.

Root diag R, 2 versions
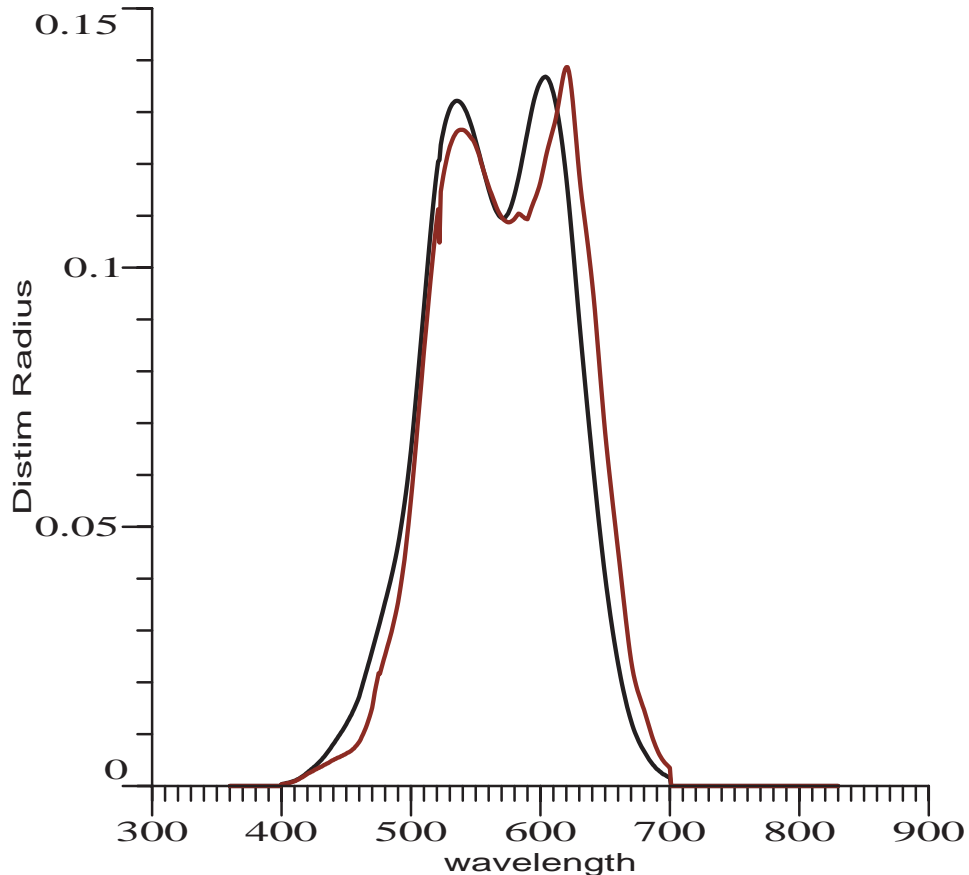
black = Root diag R for S+P red & green only.
maroon = Root diag R for S+P red + anomalous grn.

For S+P normal red & green, peaks of rdRSP =
{
[536 , 0.132176]
[571 , 0.109544]
[604 , 0.136841]
}

For S+P normal red + anomalous grn, peaks of rdRSPag =
{
[475 , 0.0217648]
[476 , 0.0216863]
[521 , 0.111223]
[522 , 0.104805]
[539 , 0.126587]
[576 , 0.108775]
[583 , 0.1104]
[590 , 0.109346]
[620 , 0.138713]
}

Again, the graphic above compares normal to green anomalous observer. Now we can look at the similar information for normal compared to red anomalous. The normal peaks should be the same.

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Root diag R, 2 versions

black = Root diag R for S+P red & green only.
maroon = Root diag R for S+P green + anomalous red.
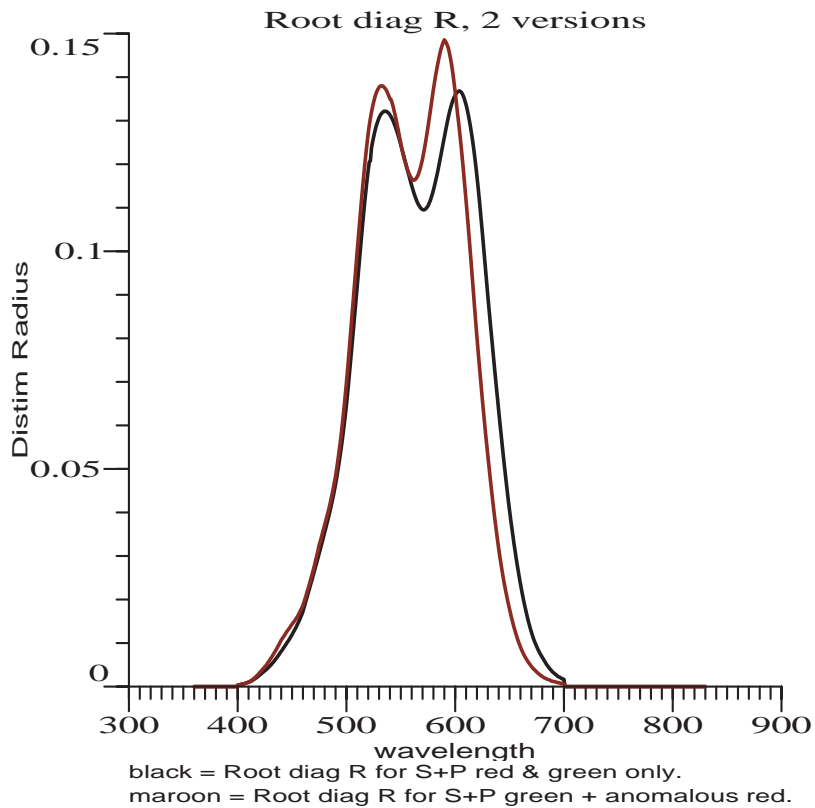
For S+P normal red & green, peaks of rdRSP =

{
[536 ,   0.13218]
[571 ,   0.10954]
[604 ,   0.13684]
}

For S+P normal grn + anomalous red, peaks of rdRSPar =

{
[532 ,   0.138]
[562 ,   0.11636]
[590 ,   0.14857]
}

Clarification: Now it may seem that I am discrediting the orthonormal basis and the locus of unit monochromats, but that is not the meaning. The scaling of the orthonormal cmf's loses the information about signal strength implicit in the simple Cornsweet plot. Instead, with the orthonormal basis, scaling relates to the stimulus. The tristimulus vector comprises the coefficients in the basis function expansion of the stimulus. See Eq. 12 of the Scottsdale article. The length (L2 norm) of the tristimulus vector is the same as that of the fundamental metamer. Scaling to the stimulus helps to separate the physics-like issues of the transduction stage from the later signal/noise and retinal processing issues.

Well, that's some food for thought. I think that the changing prime colors—as determined by the peaks of radius—may be important, but I'll stop and ask for your ideas.

Thanks
Jim Worthey
03/16/05, 01:19:43 AM

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