Camera Design Using Locus of Unit Monochromats

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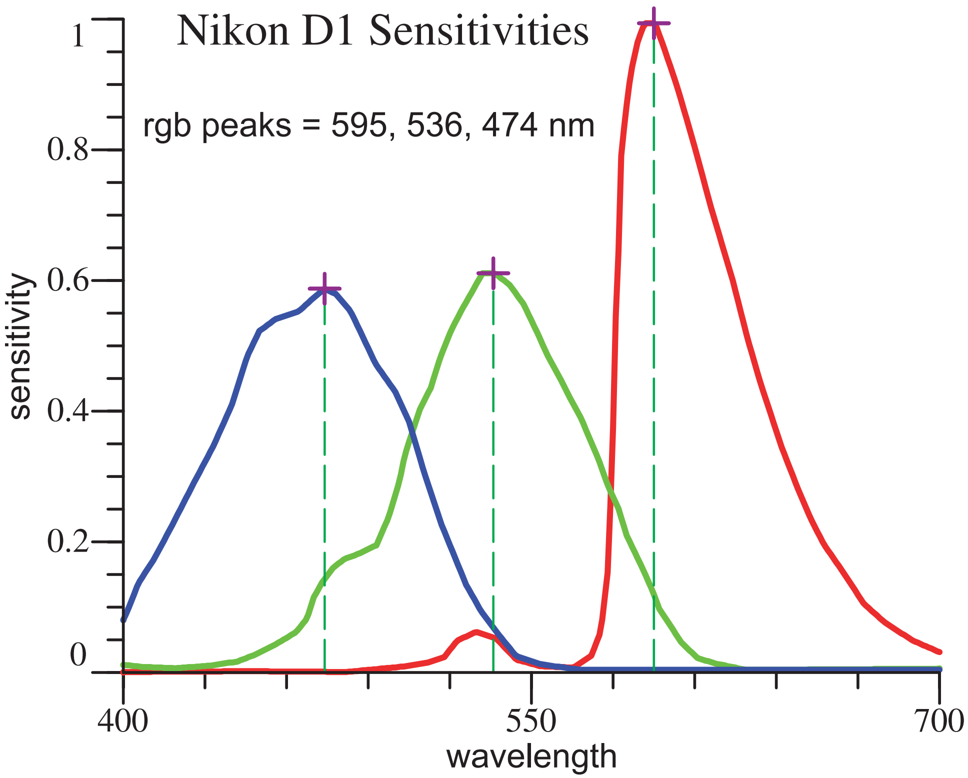
Abstract

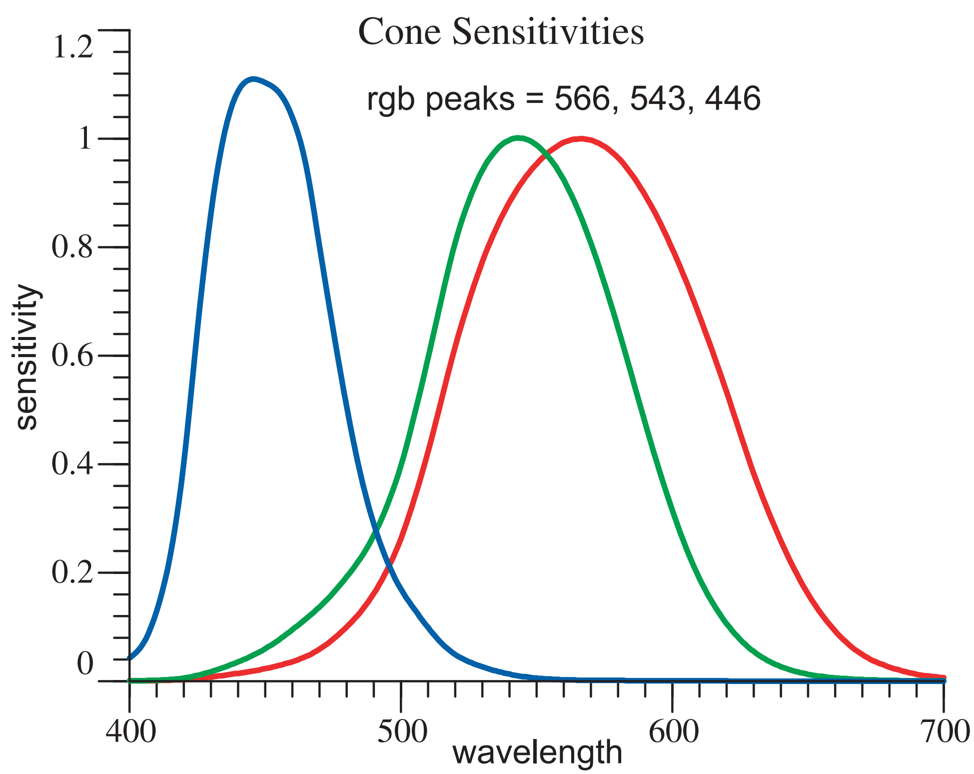
The Maxwell-Ives criterion (MI) says that for color fidelity a camera's spectral sensitivities must be linear combinations of those for the eye. W. A. Thornton found certain "prime color" (PC) wavelengths, with special importance for color vision. At CIC 6, M. H. Brill et al. spoke in favor of "cameras that have peak sensitivities at the PC wavelengths." MI and PC are related ideas. MI implies symmetry between the camera and the eye: the camera has its own prime colors, which should be similar to the eye's. At CIC 12, J. A. Worthey presented an orthonormal opponent set of color matching functions as a path to J. B. Cohen's Locus of Unit Monochromats (LUM), an invariant representation of color-matching facts. Here we present a concise method to evaluate a sensor set by comparing its LUM to the eye's. Equal LUMs would mean that MI is met, and equal PC wavelengths would tend to mean that MI is loosely met. Two sets of camera sensors can have the same LUM, but differ in the effect of sensor noise. A numerical noise example illustrates the point.

Introduction:

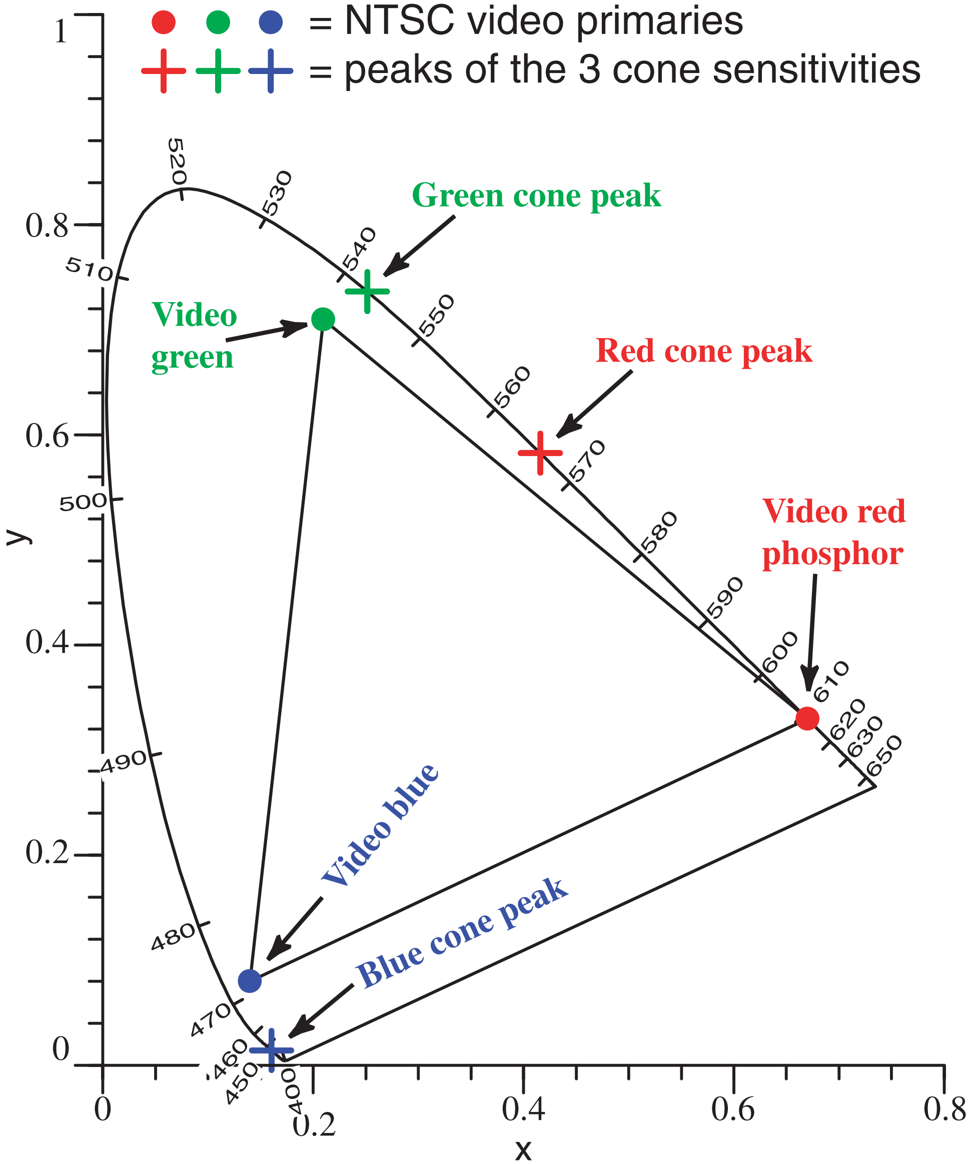
- **Maxwell-Ives**
- **Prime Colors**
- **Locus of Unit Monochromats**



Ref: DiCarlo, Montgomery & Trovinger, CIC 12
direction cosine, red & green = 0.175
direction cosine, green & blue = 0.362



The Maxwell-Ives criterion (= MI, also called the Luther criterion) teaches that, for color fidelity, the camera's 3 spectral sensitivities must be linear combinations of human color matching functions. At CIC 6, M. H. Brill *et al.* discussed applications of Prime Colors (PC) to imaging[2].



Since the prime colors are more or less the NTSC television phosphor colors[15], they must have something to do with taking and printing pictures, but does that mean that the camera sensors should peak at the prime colors? Human red cones peak at 566 nm, **not** the red prime wavelength of 603 nm.

Jozef B. Cohen derived the projection matrix R and from it the Locus of Unit Monochromats (LUM)[4-6]. At CIC 12, Worthey showed that if orthonormal color matching functions are used, vectors in Cohen's space are tristimulus vectors, a fact that Cohen probably understood, but did not emphasize. Traditional colorimetry uses tristimulus vectors, $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$, but the arbitrariness of the XYZ system obscures the meaning of color vectors.

Camera sensors are analyzed below by a simple algorithm, a few lines of computer code. It merges the criterion of Maxwell-Ives with ideas from Cohen [4-6] and elsewhere [3]:

Key Ideas

Matrix $\mathbf{R} = A(A^{T}A)^{-1}A^{T}$:

1. Is an easy method for curve-fitting. To approximate function L by a set of basis functions (the columns of A),

$$L^* = \mathbf{R}L, \qquad (1)$$

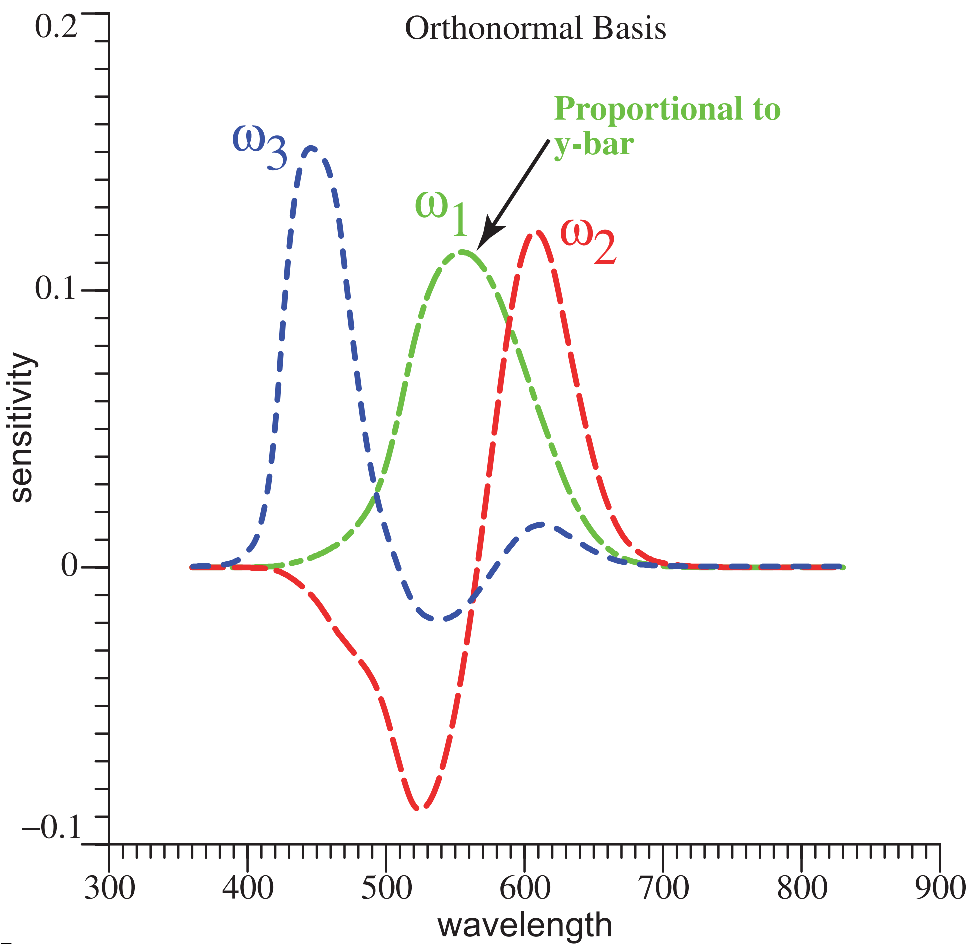
   where L^* is the least-squares best fit.
2. If L is a spectral power distribution, and the basis functions are color matching functions, then L^* is a metamer of L in the usual sense. This use of **R** has to do with understanding colorimetry, and not with analyzing noisy data.
3. If one set of color matching functions, A, is an invertible transformation of another, **R** computed from either one is the same large array of numbers.
4. The columns (or rows) of **R** create the Locus of Unit Monochromats. Therefore, by item 3, the LUM is invariant to a change of basis.

Maxwell-Ives Criterion (MI)

1. Says that a camera will have color fidelity if its sensors are linear combinations of human color matching functions.
2. Implies symmetry between the eye and the camera.
3. If a camera meets MI, then its LUM will be the same as the eye's.

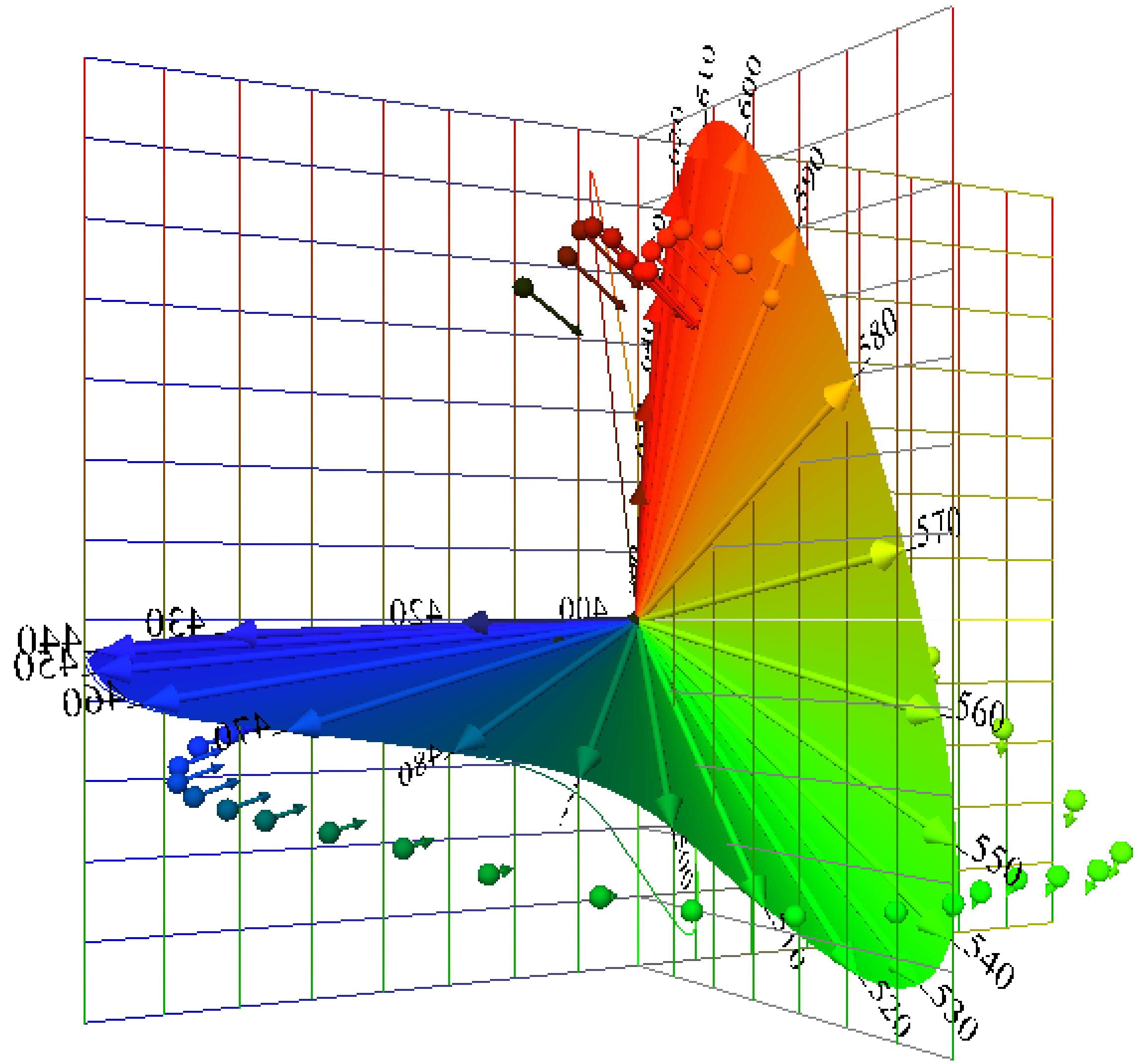
Orthonormal Basis

1. It is possible to make a set of orthonormal color matching functions that are linear combinations of another set, such as the 2° observer. The first function is the all-positive achromatic sensitivity, ω_1, proportional to the usual y-bar and a sum of red and green cones. The second, ω_2, is a red-green opponent function, with no blue-cone input. The third function, ω_3, involves all 3 cones and is a sort of blue or blue-yellow sensitivity. Grouping the orthonormal functions into one



matrix, we can write $\Omega = [|\omega_1\rangle\ \omega_2\rangle\ |\omega_3\rangle]$.

2. Combining ω_1, ω_2, ω_3 into a single 3D graph (a parametric plot) gives the Locus of Unit Monochromats. In the figure below, the edge of the surface is the human LUM. Spheres are the LUM of a Dalsa FTF3020C sensor. Arrowheads are “best fit” of camera to eye.



3. Points on the Locus of Unit Monochromats are the tristimulus vectors of narrow-band lights, plotted wavelength-by-wavelength. Each vector has a direction in color space, and an amplitude. Mixing of colors is modeled by vector addition, as in the usual XYZ system, but the orthonormal basis leads to intuitive vector

components and avoids arbitrary double-counting.

4. In short, the LUM is a detailed picture of color mixing by the eye. Its shape is invariant; the choice of orthonormal basis sets its rotation/reflection with respect to the axes.
5. A set of camera sensors has its own invariant LUM which can be found by creating an orthonormal basis. The camera's orthonormal basis can be set up for easy comparison to the eye's basis and LUM.
6. **The method of finding the camera basis, for purpose of this poster, is slightly evolved from that in the 6-page paper in the proceedings.** The evolved method is a little simpler and applies for all cameras.
7. The more evolved procedure can be called "the fit first method." **The computer code looks like this:**

```
Rcam = RCohen(rgbSens)
CamTemp = Rcam*OrthoBasis
GramSchmidt(CamTemp, CamOmega)
```

Here `rgbSens` is a matrix whose columns are the 3 camera sensor functions. `Rcam` is Cohen's projection matrix **R** based on the camera functions. `OrthoBasis` is **Ω**, the 3 orthonormal vectors for human. `CamTemp` is then the best fit to `OrthoBasis` using a linear combination of the camera sensitivities. The columns of `CamTemp` may not be orthonormal, so Gram-Schmidt finds the orthonormal basis, `CamOmega`. That's the main result, and the camera's LUM is a parametric plot of the 3 columns of `CamOmega`.

8. `CamTemp` is the same set of "fit functions" as found in the proceedings paper. Here it is found in the most expedient way, as a preliminary to finding the camera's orthonormal basis. The fit functions are in general **not orthonormal**.
9. In comparing the camera's LUM to the eye's, the essential programming is easy. Example code for routines `RCohen()` and `GramSchmidt()` is on Worthey's web site, http://www.jimworthey.com . The harder job, perhaps, is generating 3D and/or parametric graphs.

Prime Colors

1. Thornton called "Prime Colors" the 3 wavelengths that act most strongly in mixtures. [2,10] Within the LUM, the prime-color wavelengths (e.g., 446, 538, 603 nm for the 2° Standard Observer) are approximately the wavelengths of the longest tristimulus vectors (e.g., 445, 536, 604 nm). [3,13]
2. If the camera's prime color wavelengths are similar to the eye's, that is at least weak conformance to the Maxwell-Ives goal.

Fun with Orthonormal Functions

1. This poster emphasizes the Maxwell-Ives criterion and its realization as a graphical comparison between the camera's LUM and that of the eye. Beyond that, the method is open-ended. Color mixing properties of the eye and of the camera have been expressed in a rationalized form (the LUMs). Orthonormal representations are easy to work with.
2. For example, the paper has a worked signal-to-noise example of two cameras that give the same signal, but with differing amounts of noise. Derivations within the example are simplified because of the orthonormal basis.

3. Many derivations can be done with a unity operator, $\Omega\Omega^{T}$. When the orthonormal basis Ω has been found, then $\mathbf{R} = \Omega\Omega^{T}$. Rather than multiply out $\Omega\Omega^{T}$ to get $\mathbf{R}$, we can use the unity operator differently. Suppose that we want to know the conversion from tristimulus vectors in the orthonormal scheme to those in the old-fashioned XYZ system. The tristimulus vectors are 3-vectors, but we can't start there.

4. Let's say that the XYZ basis is $A = [|\bar{x}\rangle \ \ |\bar{y}\rangle \ \ |\bar{z}\rangle]$, and Ω is an orthonormal basis also based on the 2° observer. Then the projection operation on A does not change it.

$$A = \Omega\Omega^{T} A \quad . \qquad (2)$$

Now group terms:

$$A = \Omega[\Omega^{T} A] \quad . \quad (3)$$

The product in square brackets is a 3×3 matrix. Give it a name:

$$B = [\Omega^{T} A] \quad . \qquad (4)$$

Then

$$A = \Omega B \quad . \qquad (5)$$

Now if $|L\rangle$ is a light's SPD expressed as a column vector, and the XYZ tristimulus vector is called Z, then the usual calculation can be expressed as:

$$Z = A^{T}|L\rangle \qquad . \qquad (6)$$

In the orthonormal system, the tristimulus vector is called V:

$$V = \Omega^{T}|L\rangle \qquad . \qquad (7)$$

So, combining Eqs. (6) and (7), etc:

$$Z = (\Omega B)^{T}|L\rangle \qquad\qquad (8)$$

$$Z = B^{\mathrm{T}}\,\Omega^{\mathrm{T}}|L\rangle \qquad (9)$$

$$Z = B^{\mathrm{T}}V \quad , \quad (10)$$

which is the transformation that was sought, from tristimulus vector V in the orthonormal system to tristimulus vector Z in the legacy system.

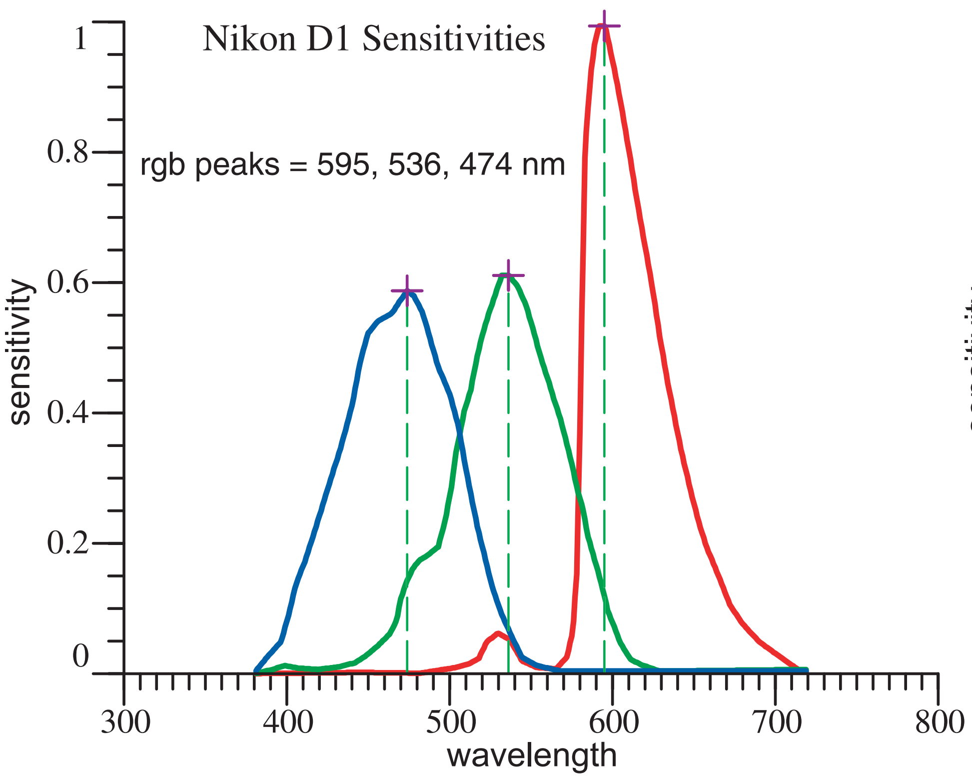
5. The essential trick is in Eq. (2) where $\Omega\Omega^{\mathrm{T}}$ does not change A, because the columns of A are linear combinations of the orthonormal basis Ω. With those ideas in mind, one can derive needed formulas, such as the transform from camera signal (a 3-vector) to a vector in the camera's orthonormal or best fit system (2 different possibilities!).
6. **Figure of merit**: The methodology presented puts the eye's and camera's color matching information into a standardized form. By extension, the "best fit" functions for the camera are in a standard form, since they are a best fit to Ω, the eye's standardized functions. We can let Δ be the discrepancy matrix, and Φ be the matrix of fit functions as in the proceedings. Then

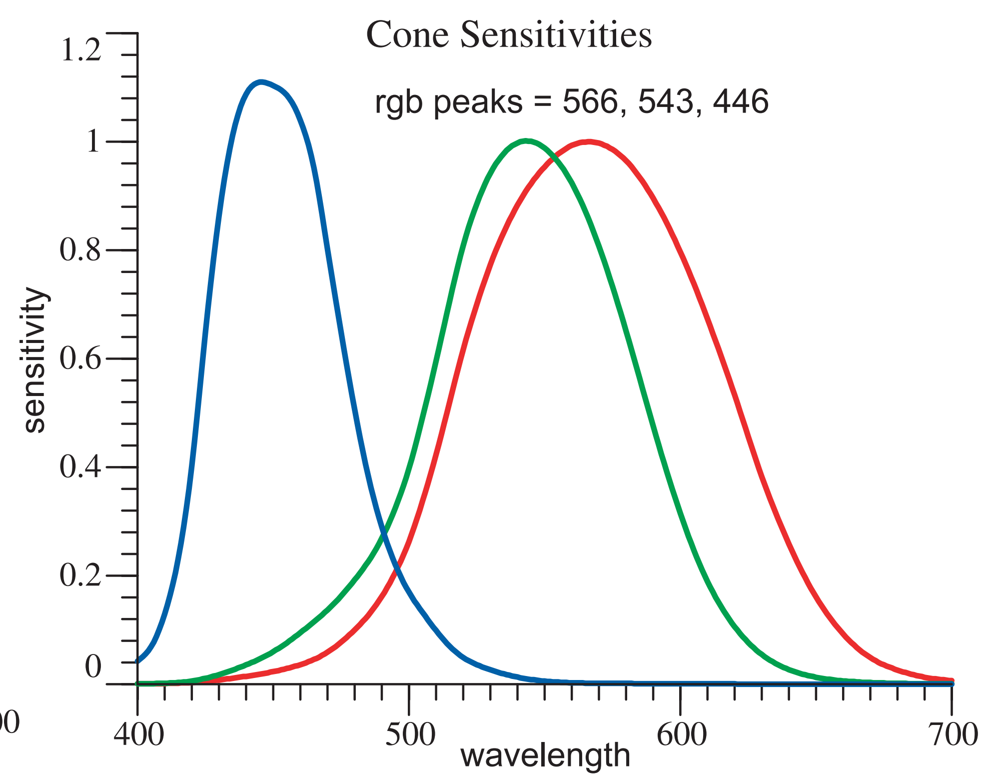
$$\Delta = \Phi - \Omega \quad . \quad (13)$$

Each row of Δ is a vector difference showing the error of the best-fit functions at a particular wavelength. The summed square of those 3 elements is the squared vectorial error. The sum of the wavelength-by-wavelength errors is the total sum-square error of the approximation, and a suitable figure of merit.

Examples:

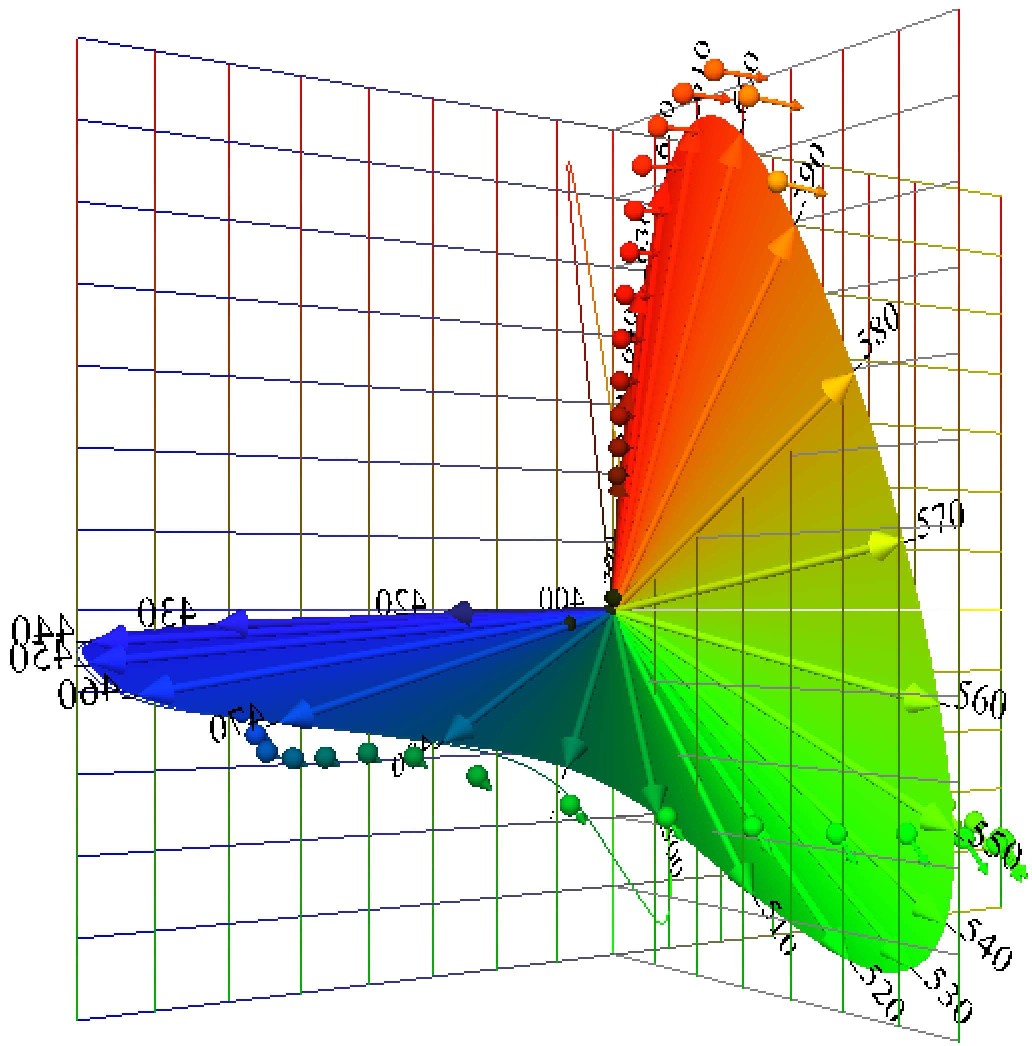
Nikon D1 Camera



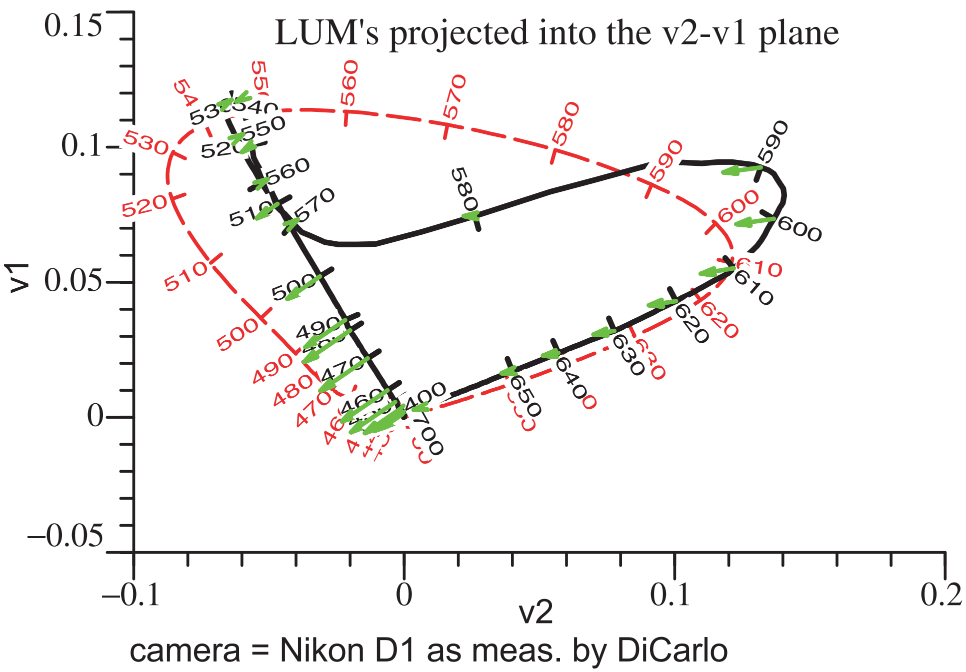


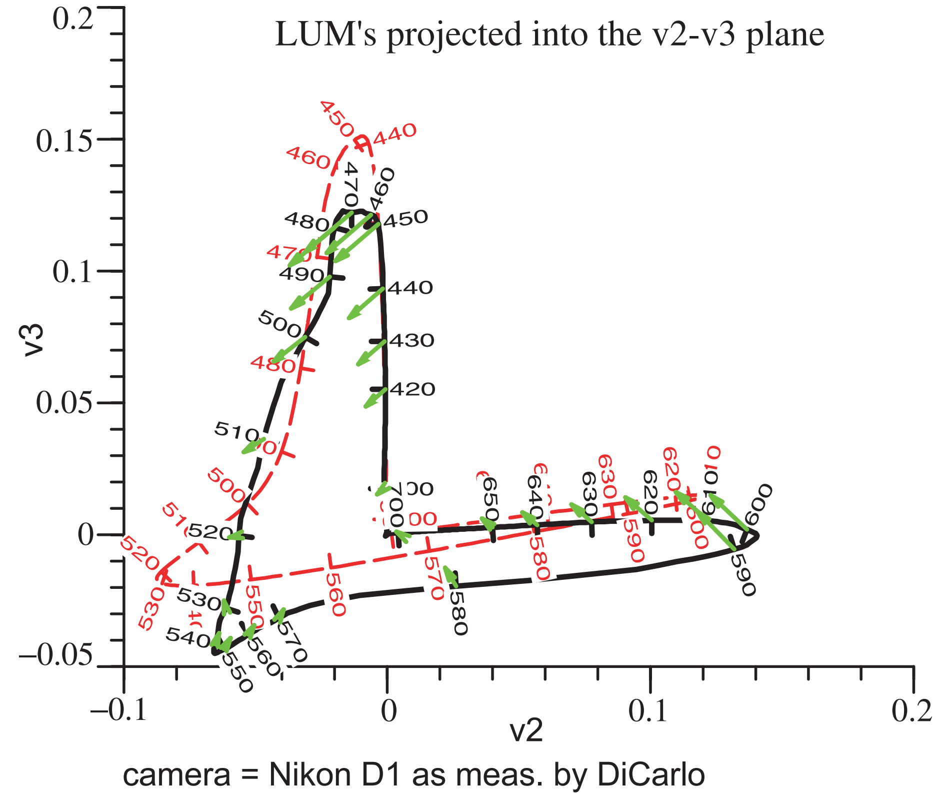
The camera sensors can be compared to human cones. The smooth curve below, shown as the edge of a surface, is the human LUM. Spheres are the camera LUM according to the *fit first* method. Arrowheads are the best fit of camera sensors to human. The same info is below in other formats.

Nikon D1 Camera (continued)



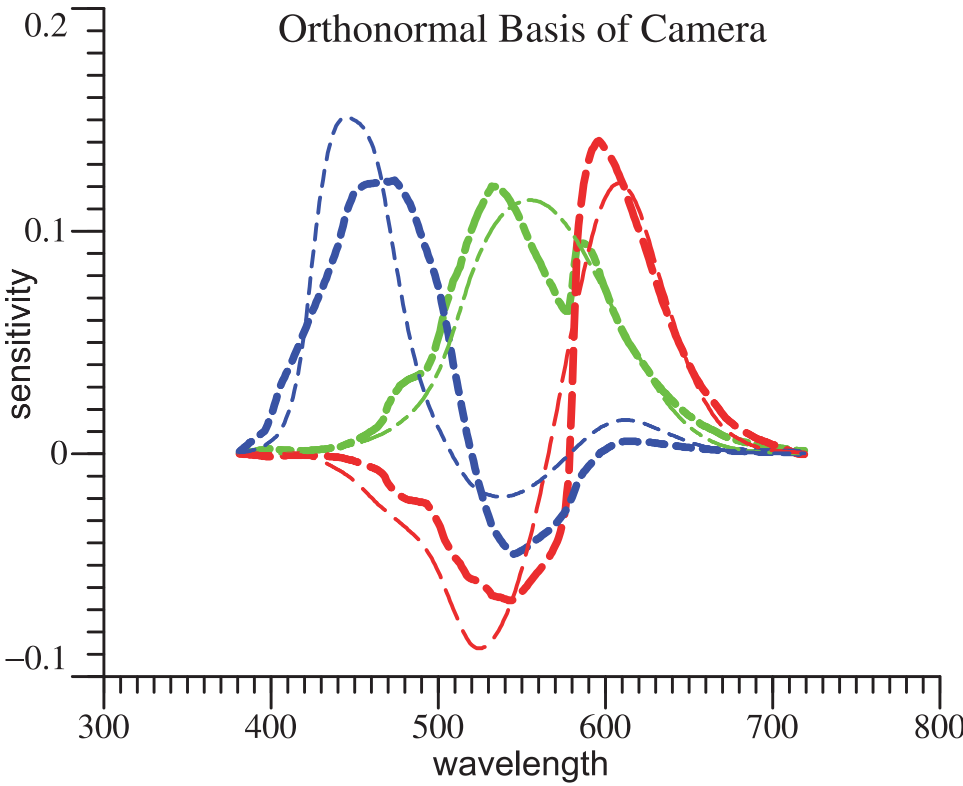
Nikon D1 Camera (continued)



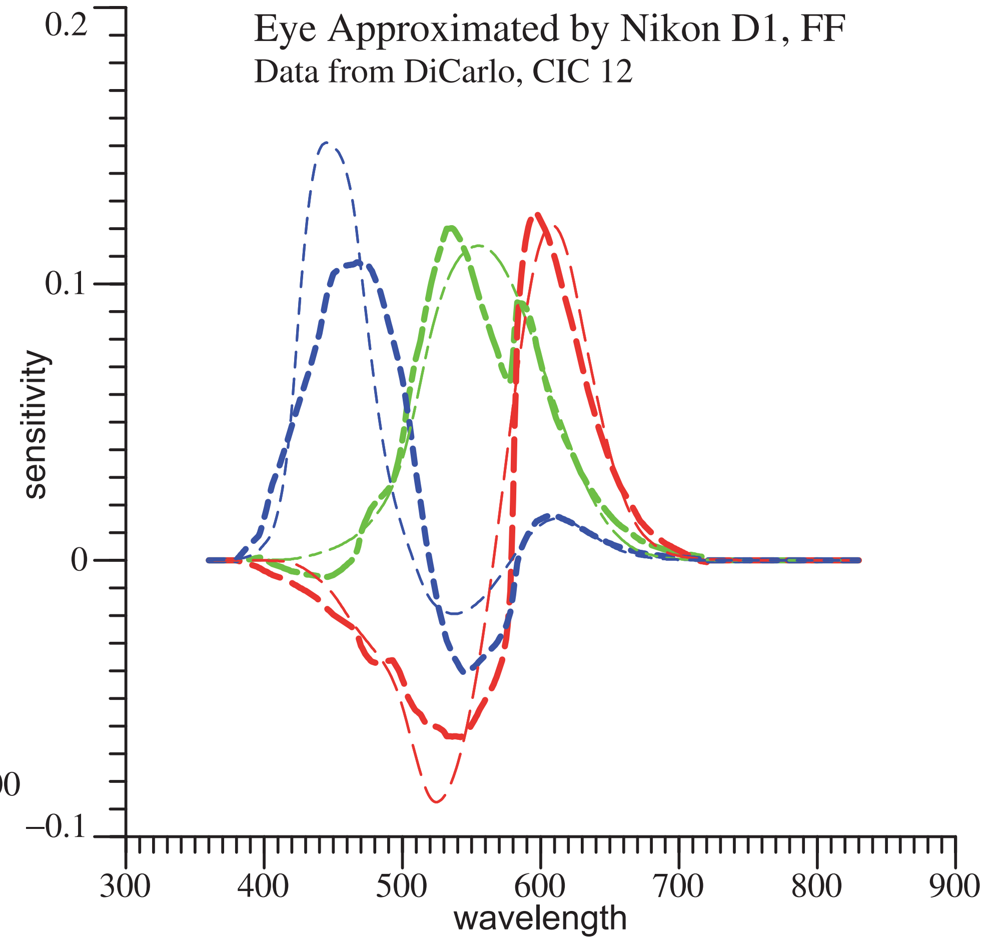


Each of the figures above is a 2D projection of the eye's LUM (red dashes), the camera LUM (black solid), and the best fit (green arrowheads). The camera designer might want to see the same information in ordinary graphs versus wavelength:

Nikon D1 Camera (continued)

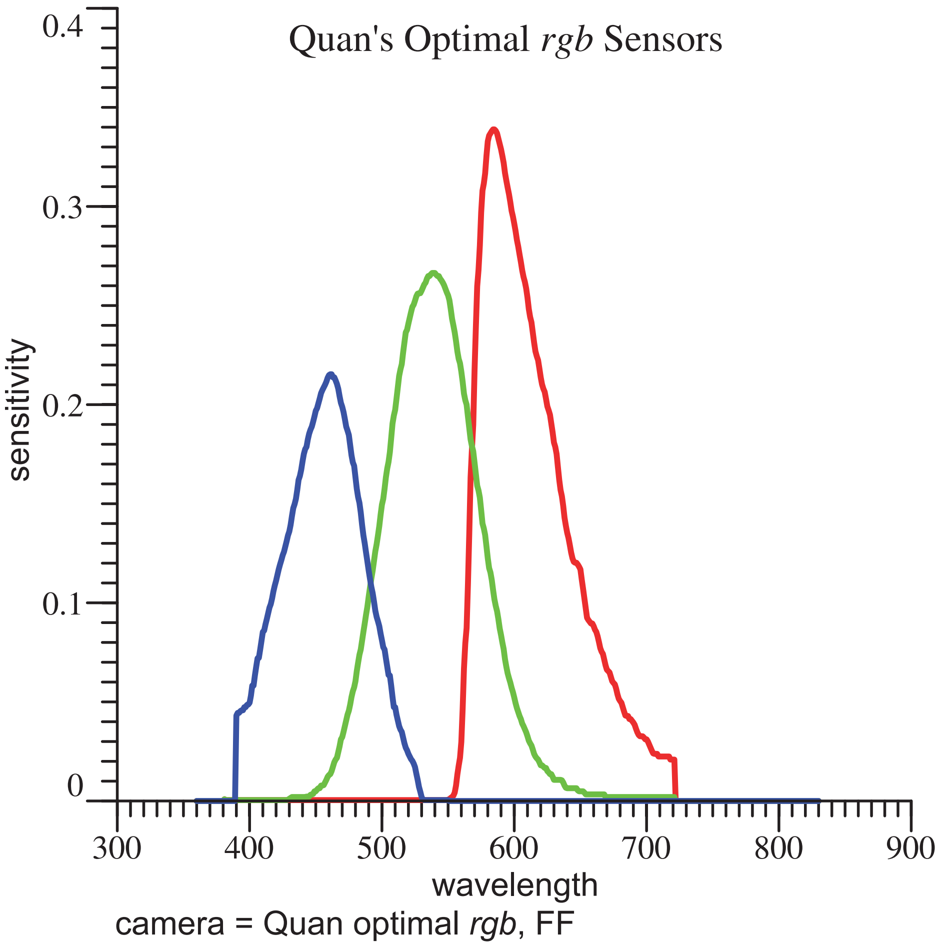


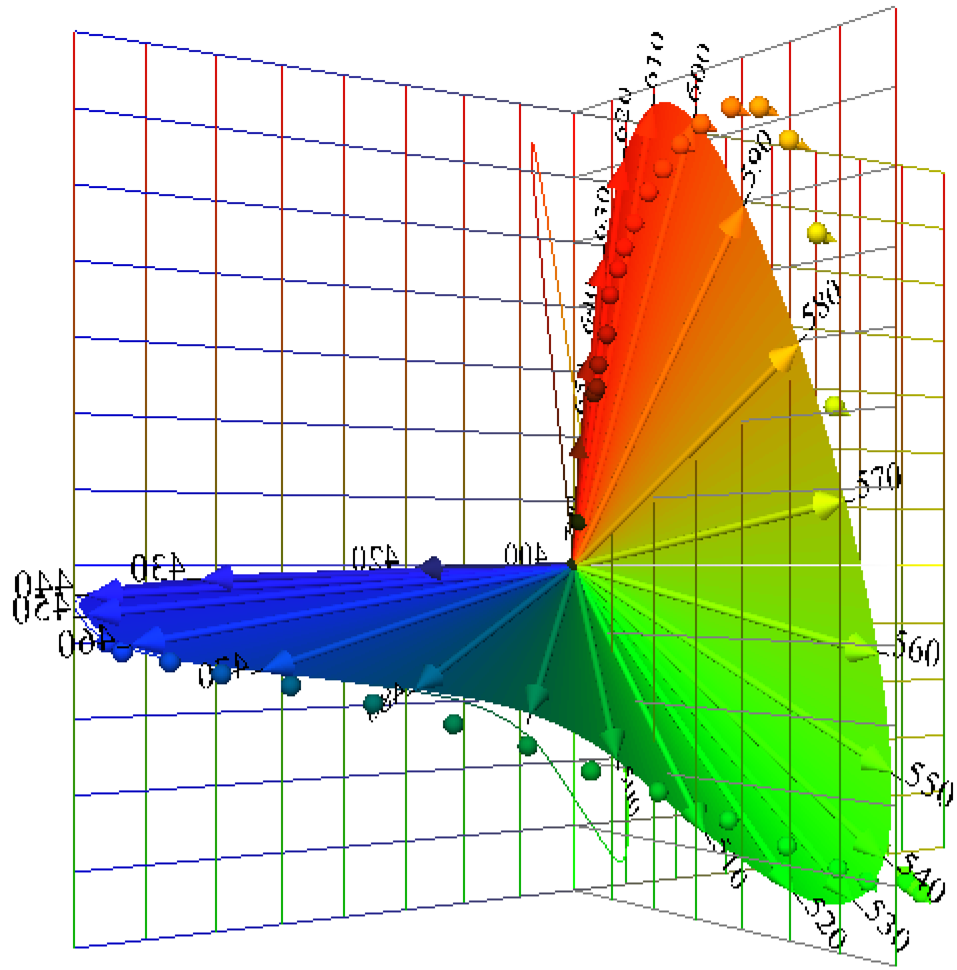
thicker curves = camera data from DiCarlo
thinner = 2 degree observer
long-short = ω1, long dashes = ω2, short dashes = ω3



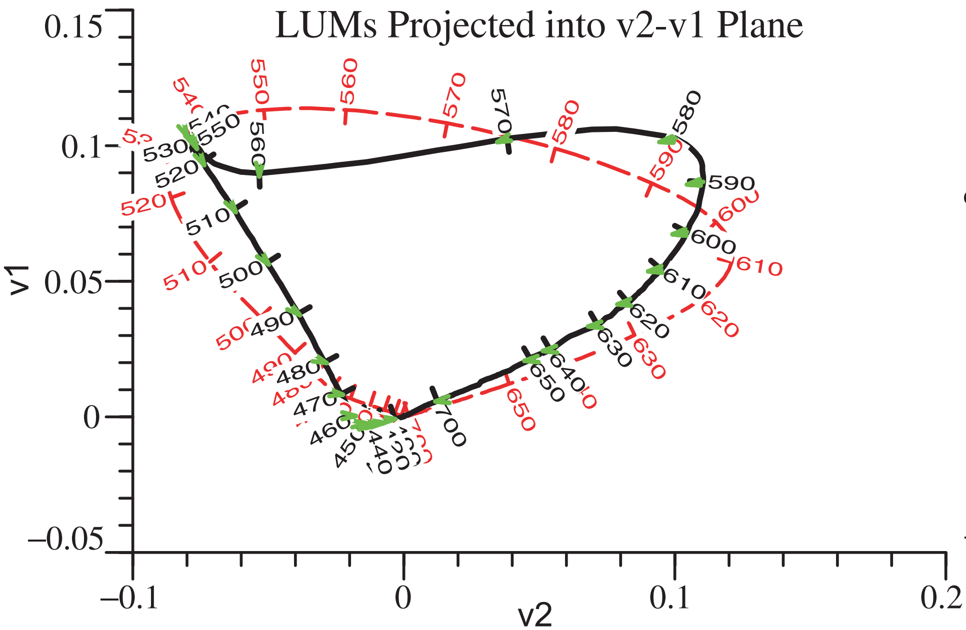
Thin = 2 deg Observer, orthonormalized
Thicker = 2 deg Observer approximated by Nikon D1
long-short = ω1, long dashes = ω2, short dashes = ω3
Fit first, then orthonormalized, then fit again.
The second fit should be the same as the first, but
for orderly software, the fit is repeated.

Quan's Optimal Sensors

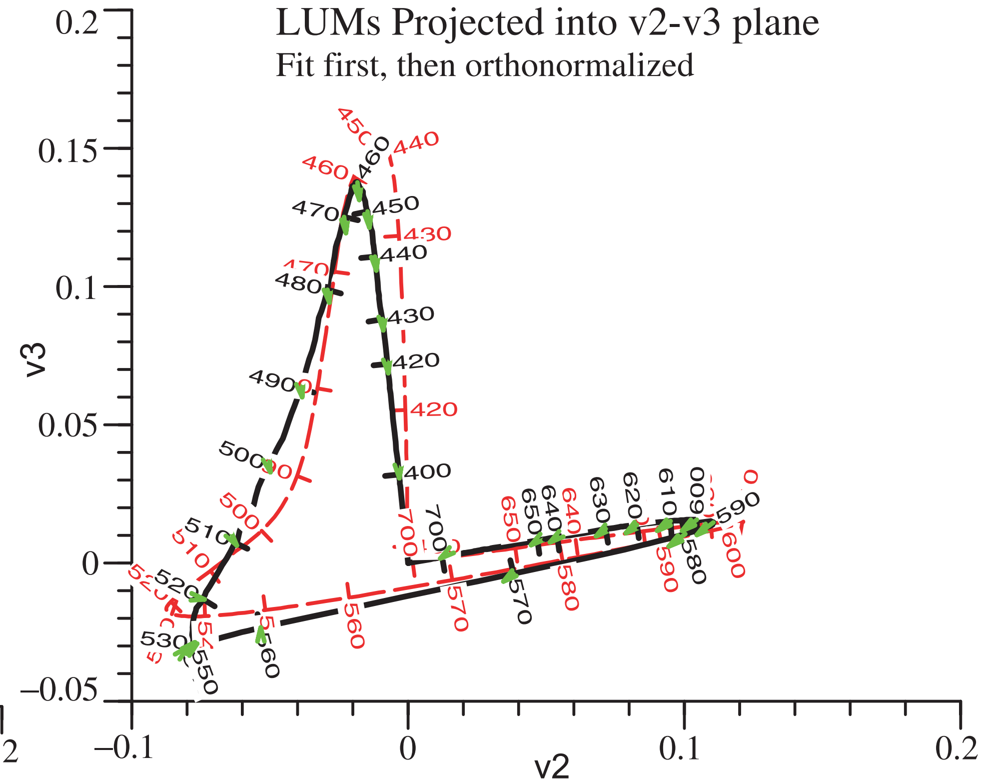




Quan’s Optimal Sensors (continued)

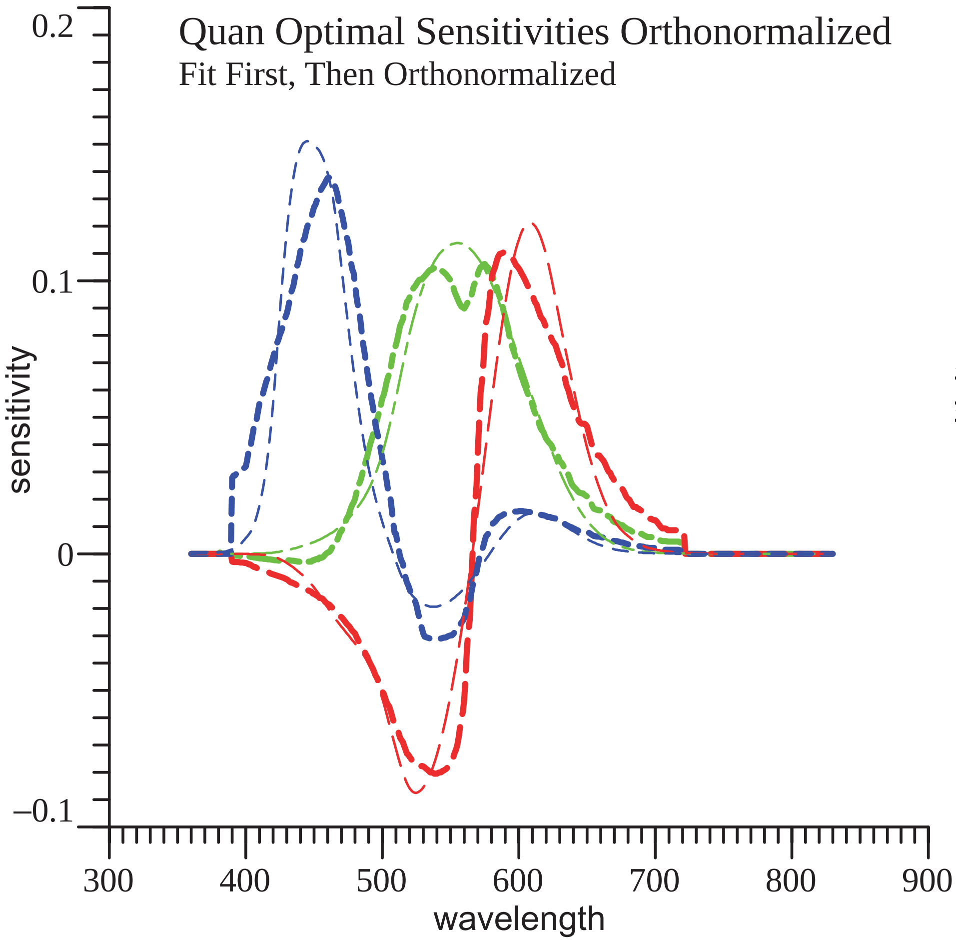


camera = Quan Optimal *rgb*,FF method
Fit first, then orthonormalized
red dashes = LUM of 2 degree observer
black solid = LUM of Quan Optimal *rgb*, fit first method
heads of green arrows = best fit to 2 deg obs by *rgb*

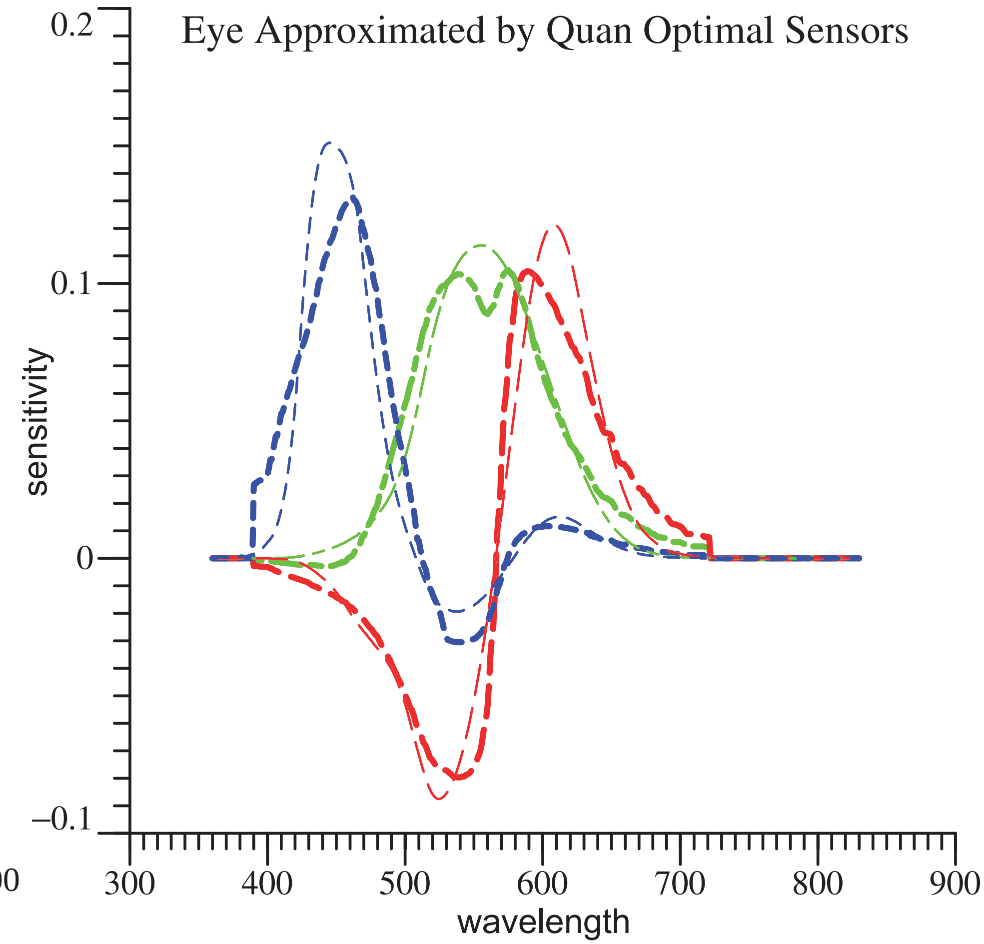


camera = Quan's Optimal *rgb*, FF method
red dashes = LUM of 2 degree observer
black solid = LUM of Quan's *rgb*, fit first method
heads of green arrows = best fit to 2 deg obs by camera

Quan's Optimal Sensors (continued)

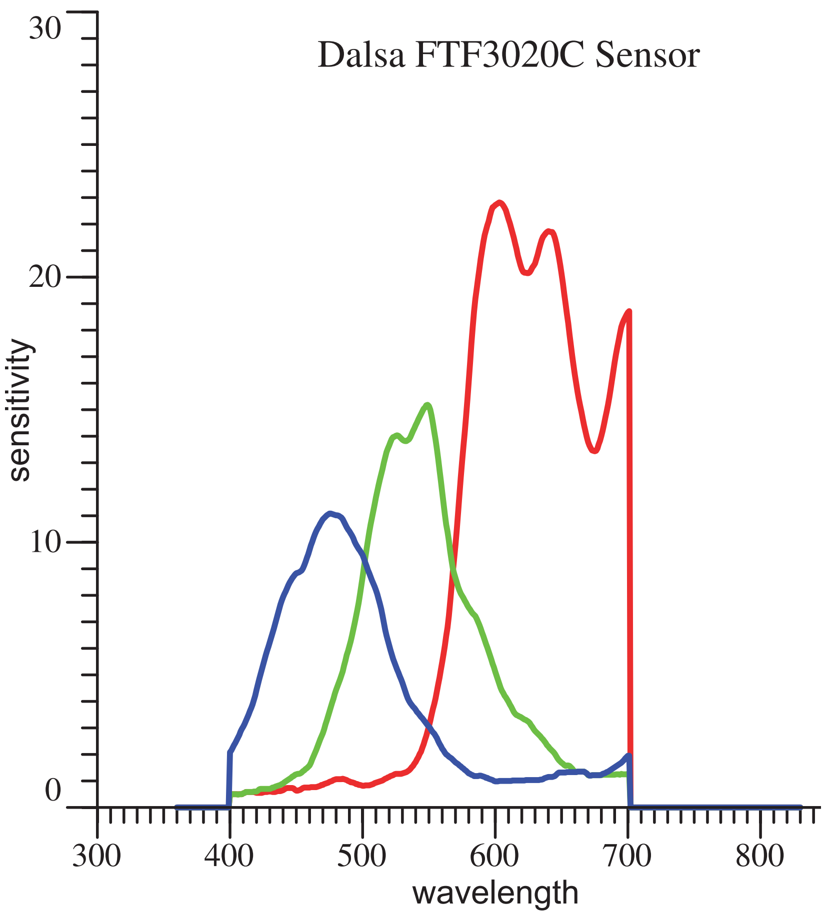


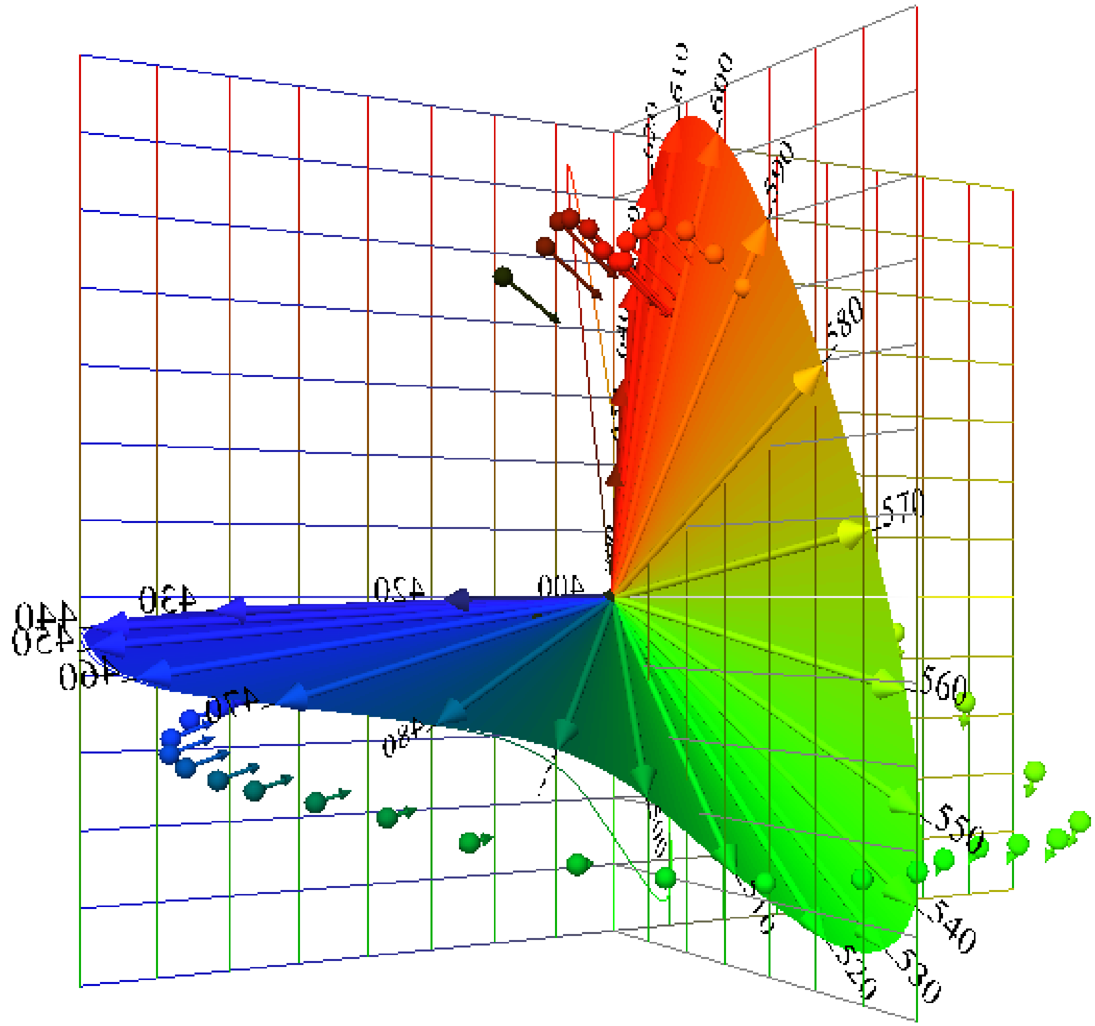
thicker curves = Quan optimal *rgb*, FF method
thinner = 2 degree observer
long-short = ω1, long dashes = ω2, short dashes = ω3



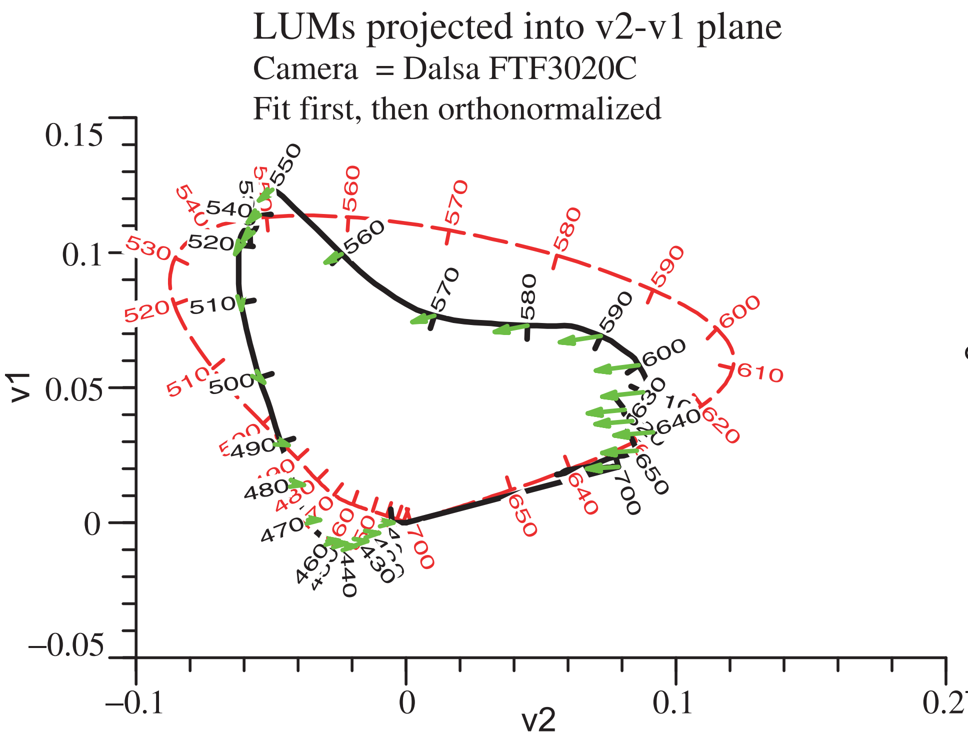
Thin = 2 deg Observer, orthonormalized
Thicker = 2 deg Obs. approx. by Quan optimal sensors
Fit first, then orthonormalized, then fit again.
The second fit should be the same as the first, but
for orderly software, the fit is repeated.
long-short = ω1, long dashes = ω2, short dashes = ω3

Dalsa FTF3020C Sensor

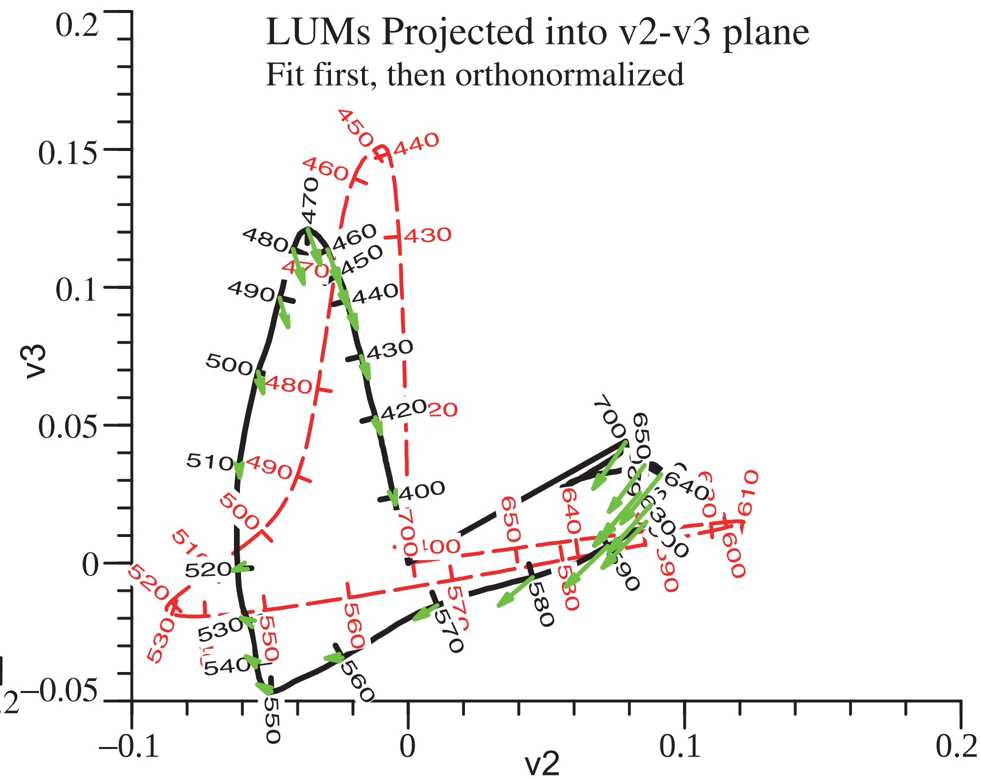




Dalsa FTF3020C Sensor (continued)

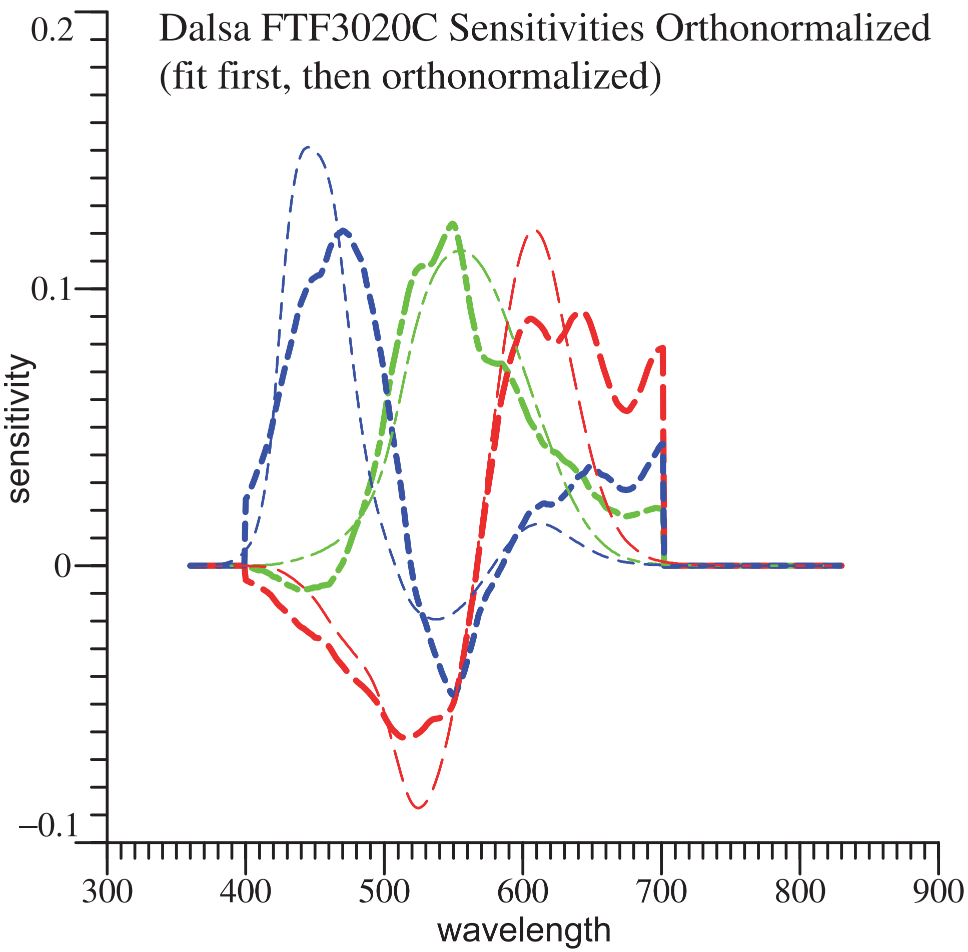


camera = Dalsa FTF3020C

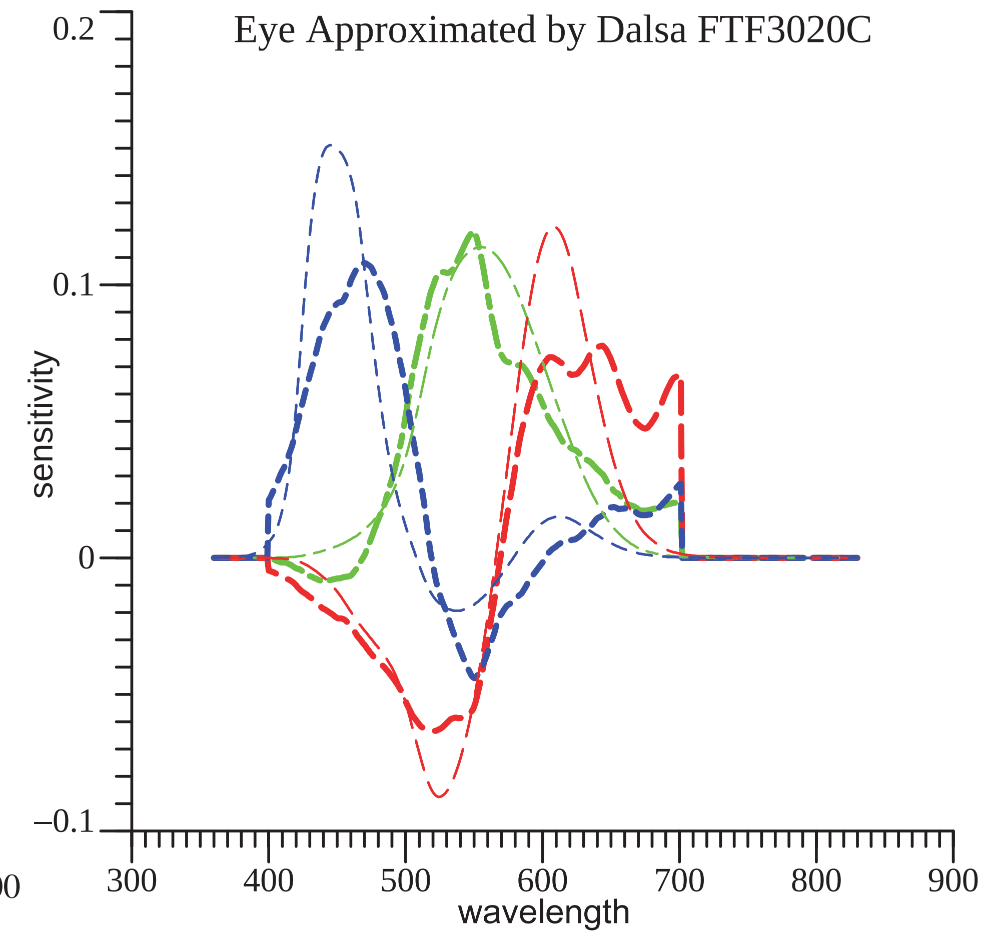


camera = Dalsa FTF3020C, FF method
Fit first, then orthonormalized
red dashes = LUM of 2 degree observer
black solid = LUM of Dalsa FTF3020C, fit first method
heads of green arrows = best fit to 2 deg obs by camera

Dalsa FTF3020C Sensor (continued)

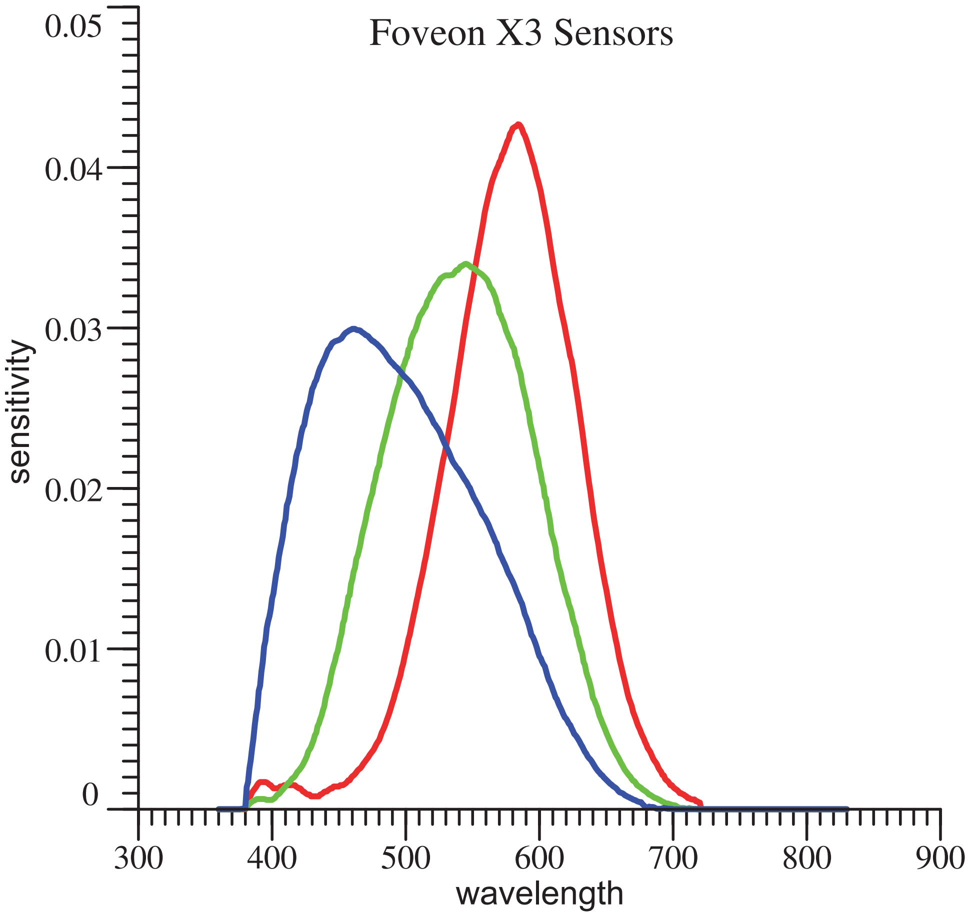


thicker curves = Dalsa FTF3020C sensitivities, FF
thinner = 2 degree observer
long-short = ω1, long dashes = ω2, short dashes = ω3
Camera sens. fit first, then orthonormalized.

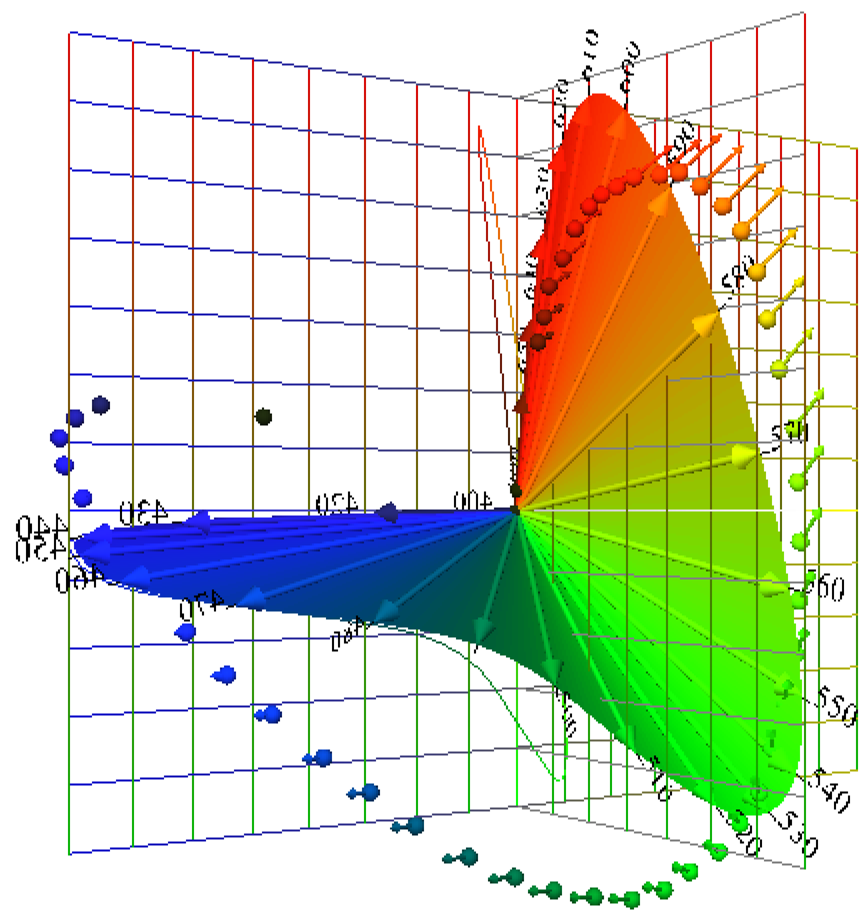


Thin = 2 deg Observer, orthonormalized
Thicker = 2 deg Observer approx. by Dalsa FTF3020C
long-short = ω1, long dashes = ω2, short dashes = ω3
Fit first, then orthonormalized, then fit again.
The second fit should be the same as the first, but
for orderly software, the fit is repeated.

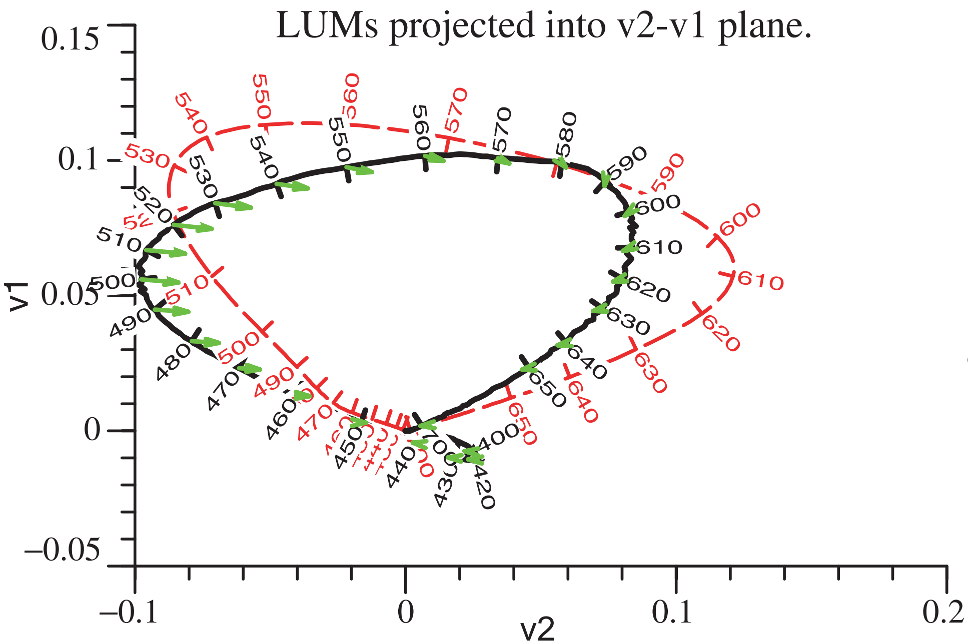
Foveon X3 Sensors, **Without** Prefilter



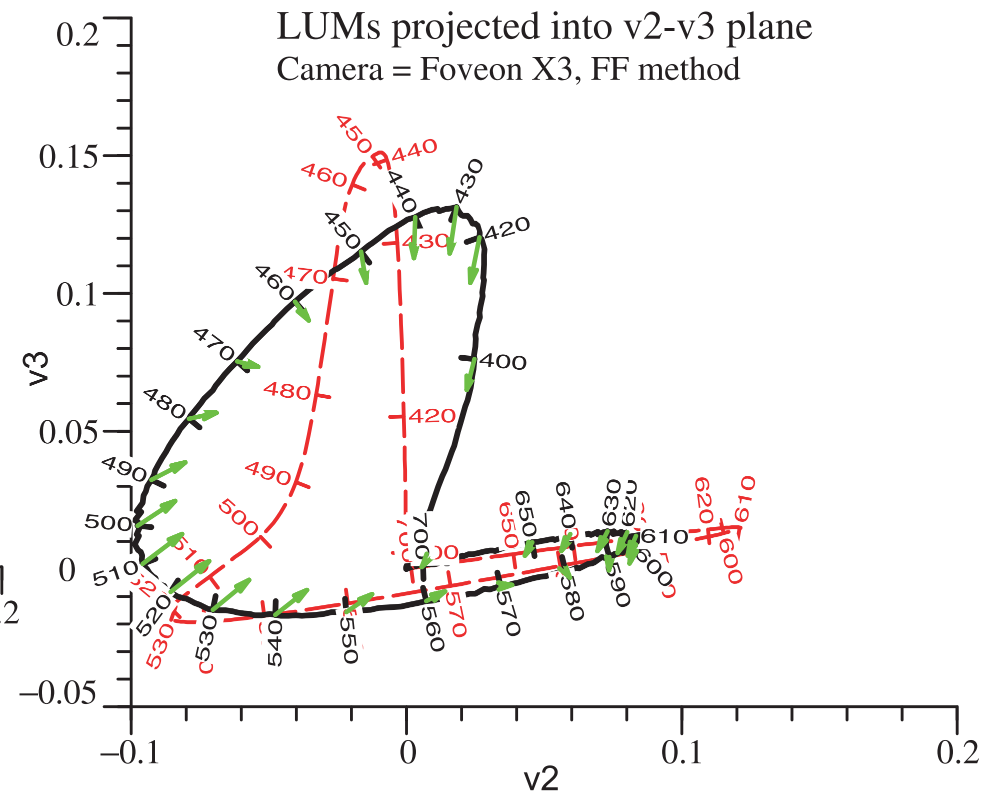
Ref: Lyon & Hubel, "Eyeing the camera: into the next century," CIC 10
camera = FoveonX3, FF



Foveon X3 Sensors, **Without** Prefilter (continued)

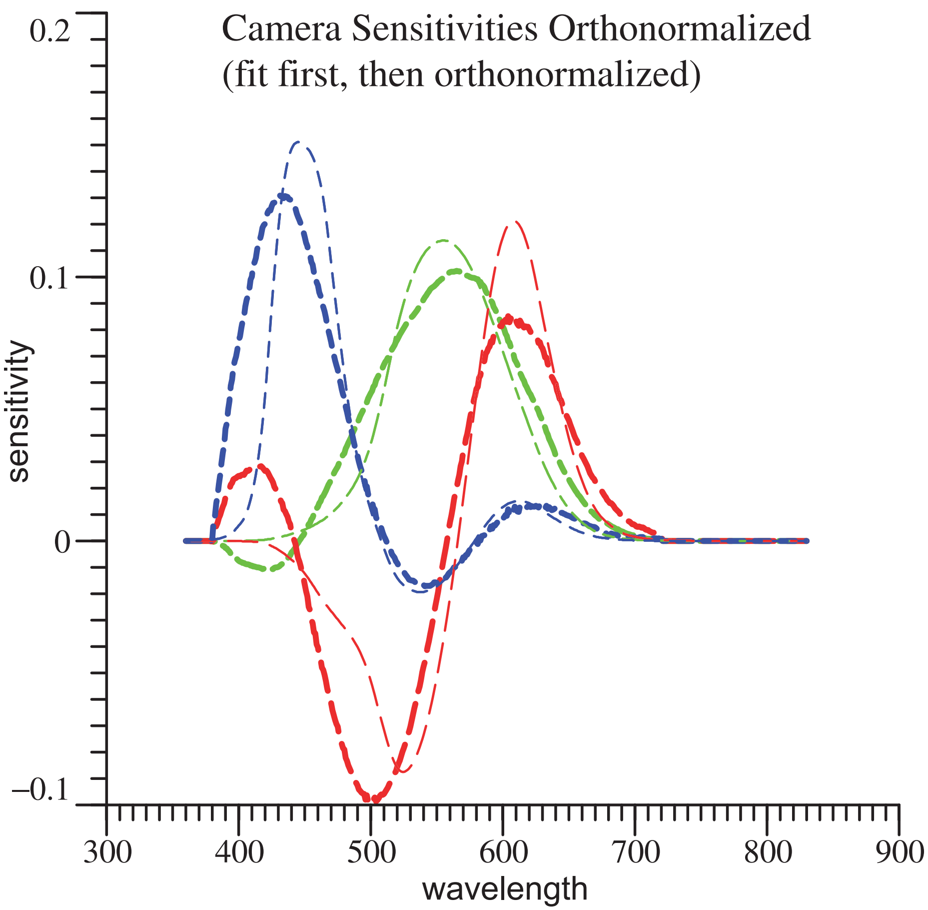


Ref: Lyon & Hubel, "Eyeing the camera: into the next century," CIC 10
Fit first, then orthonormalize.
Red dashes = human LUM
Black solid = camera LUM, Foveon X3, FF method

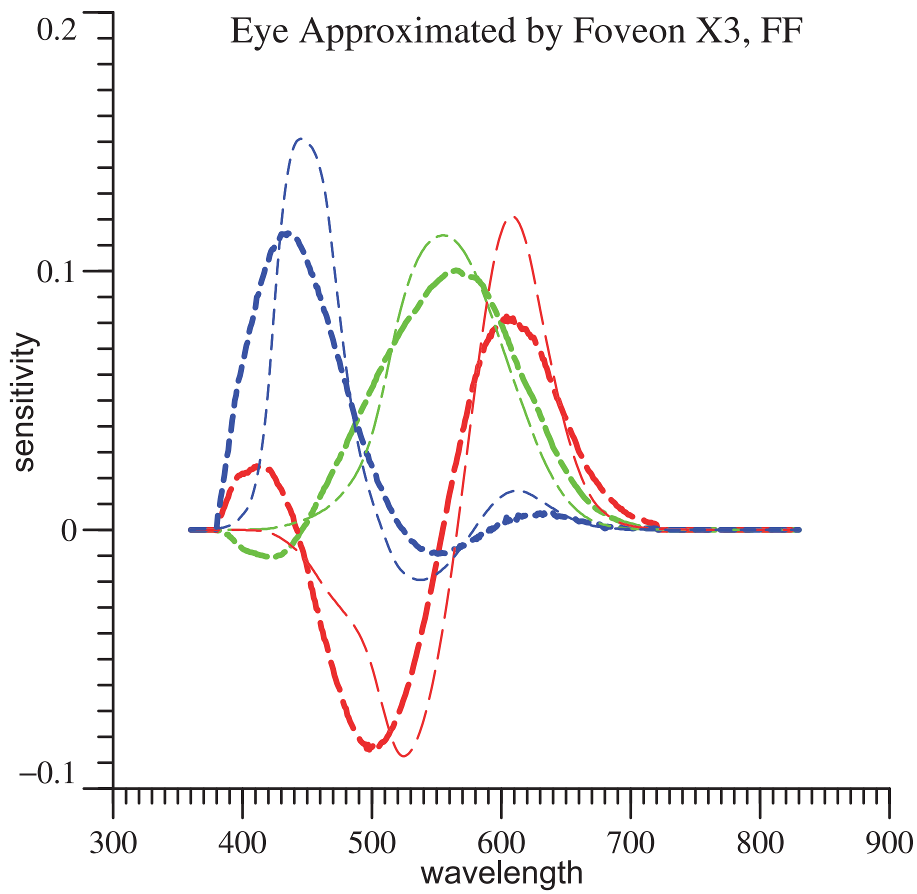


Camera = Foveon X3, FF method
Ref: Lyon & Hubel, "Eyeing the camera: into the next century," CIC 10
Fit first, then orthonormalize.

Foveon X3 Sensors, **Without** Prefilter (continued)

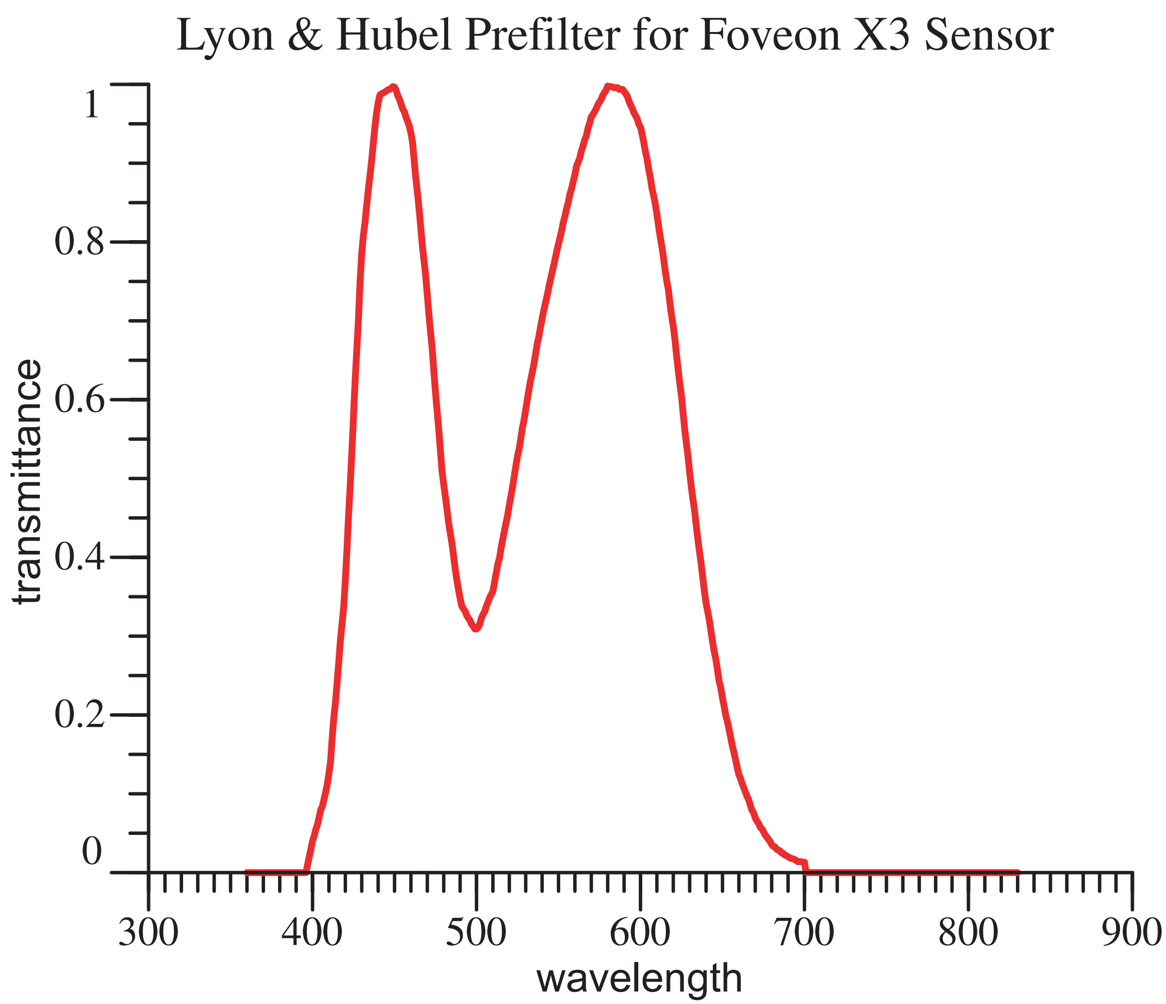


thicker curves = Foveon X3 sensitivities, FF
thinner = 2 degree observer
long-short = ω1, long dashes = ω2, short dashes = ω3
Camera sens. fit first, then orthonormalized.



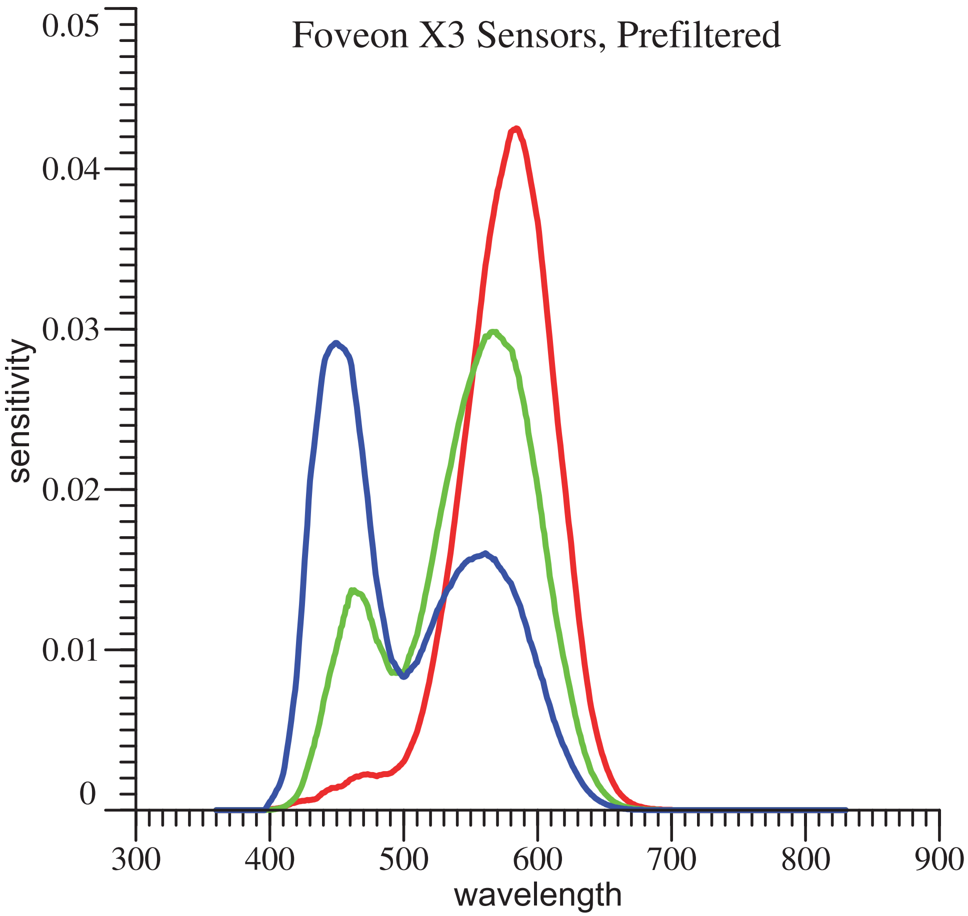
Thin = 2 deg Observer, orthonormalized
Thicker = 2 deg Observer approximated by Foveon X3
Fit first, then orthonormalized, then fit again.
The second fit should be the same as the first, but
for orderly software, the fit is repeated.

Lyon and Hubel propose a “prefilter” over the Foveon X3 array:

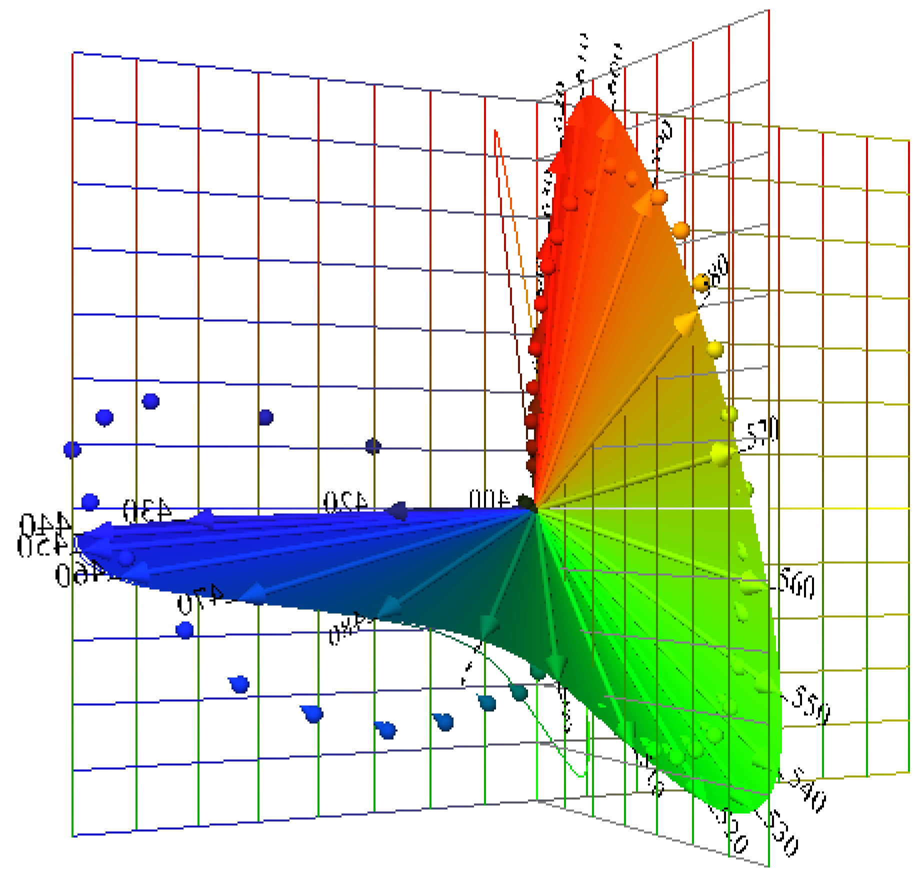


On the next page, the prefilter is combined with the sensors above.

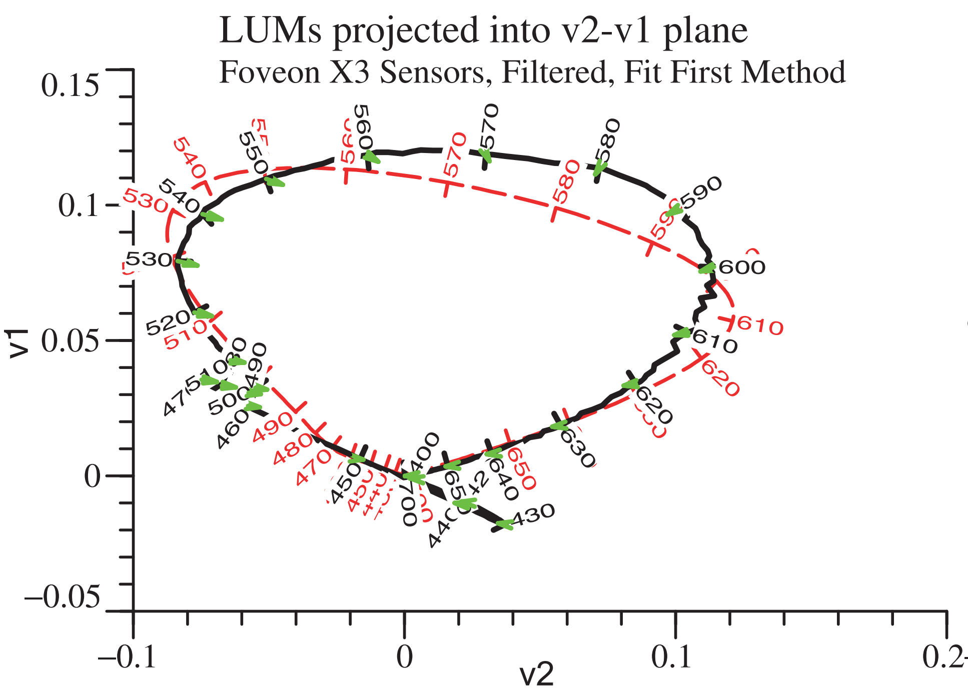
Foveon X3 Sensors, **With** Prefilter



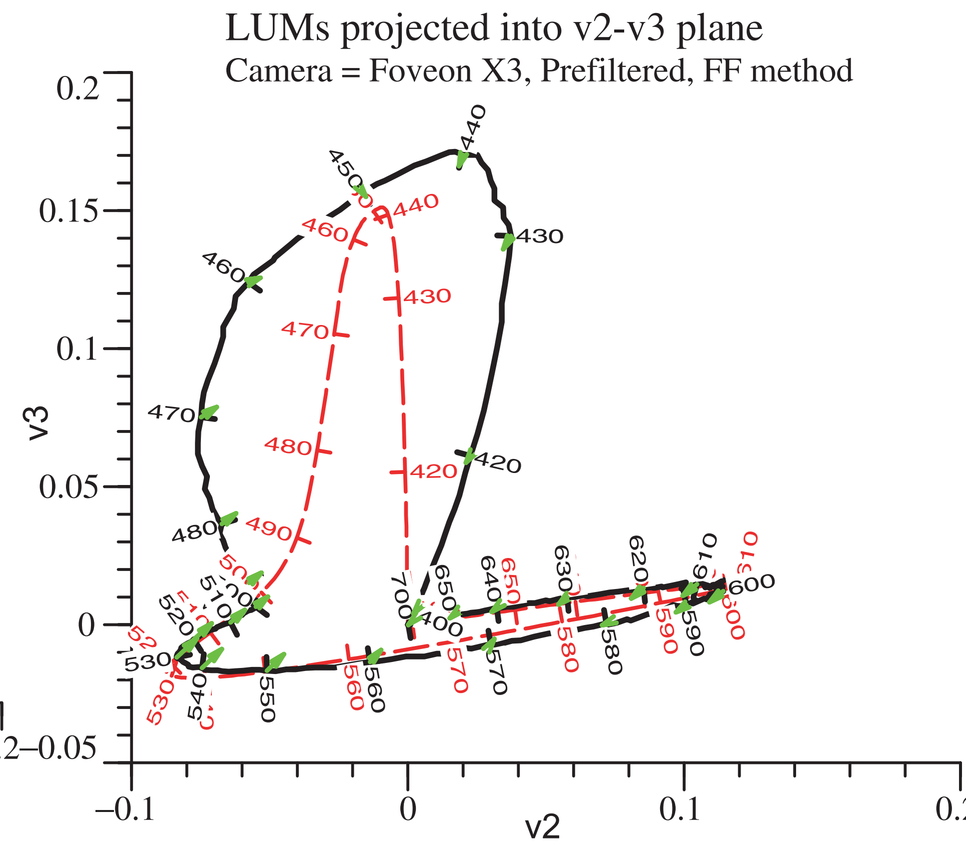
Ref: Lyon & Hubel, "Eyeing the camera: into the next century," CIC 10
camera = FoveonX3, Prefiltered, FF



Foveon X3 Sensors, **With** Prefilter (continued)

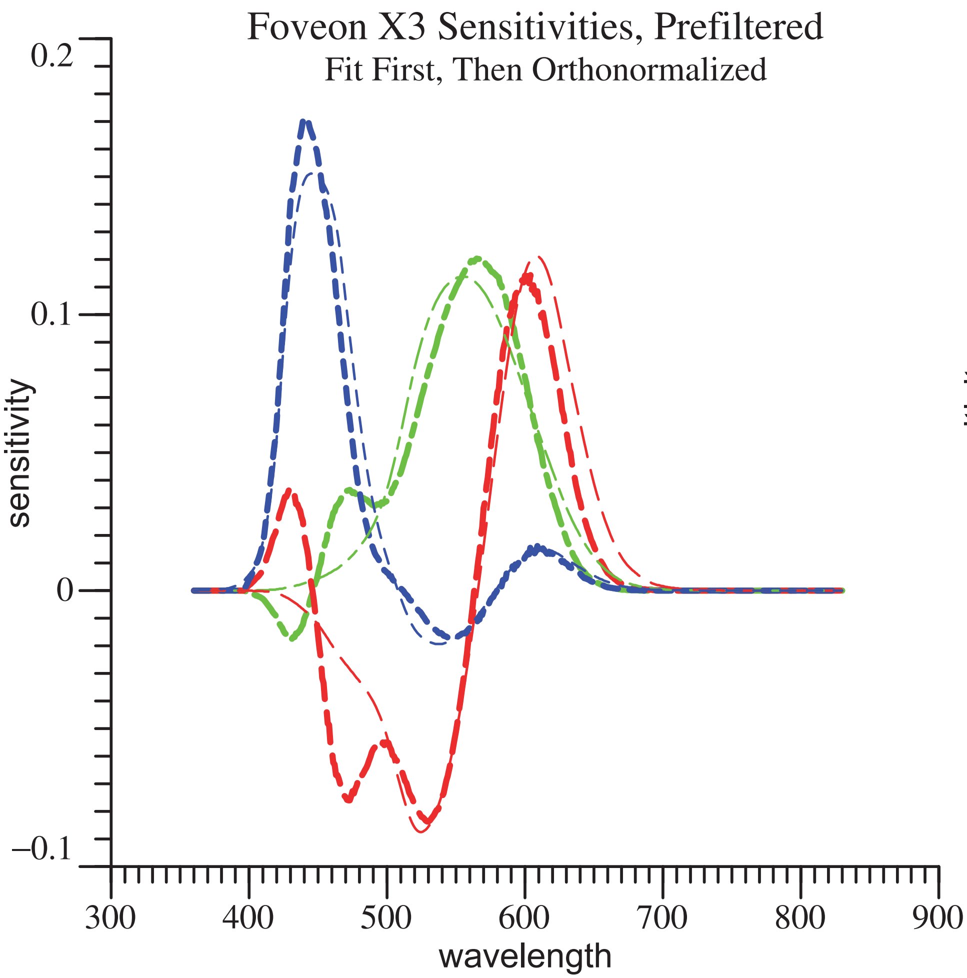


Ref: Lyon & Hubel, "Eyeing the camera: into the next century," CIC 10
Fit first, then orthonormalize.
Red dashes = human LUM.
Black solid = camera LUM, Foveon X3 Prefiltered, FF method.

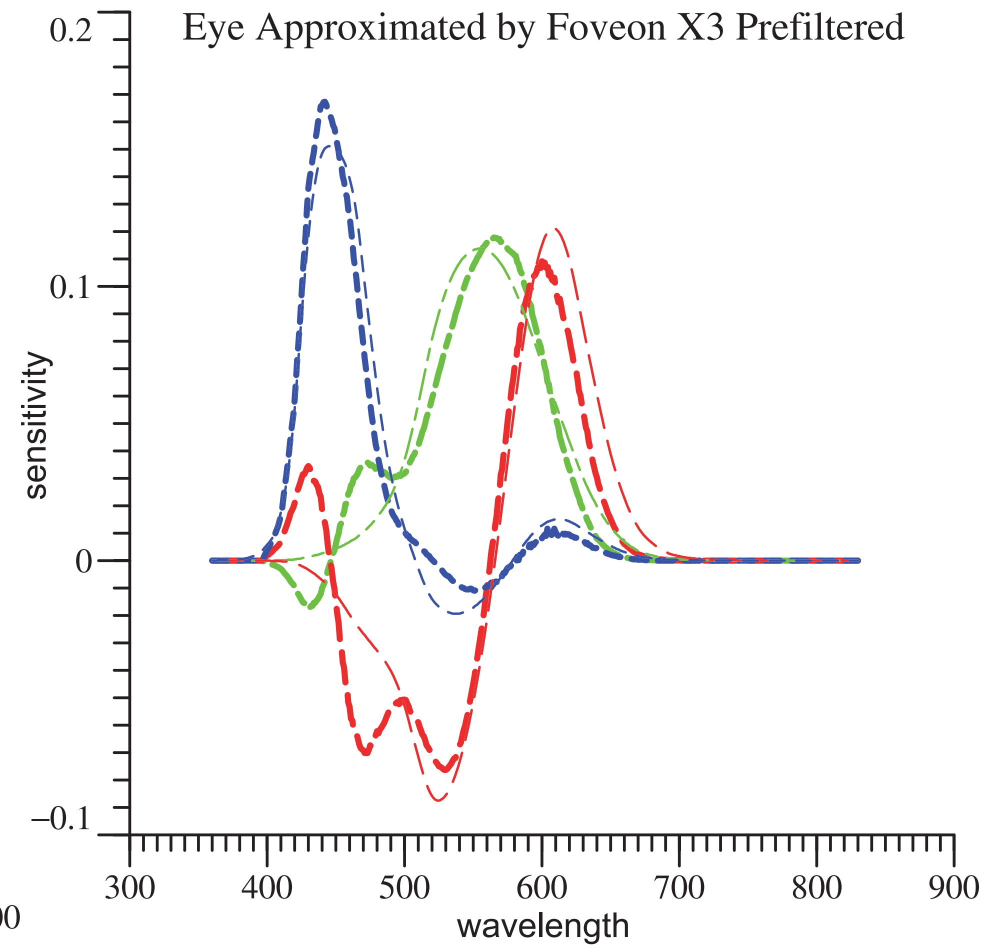


Ref: Lyon & Hubel, "Eyeing the camera: into the next century," CIC 10
Fit first, then orthonormalize.
Red dashes = human LUM.
Black solid = camera LUM, Foveon X3, Prefiltered, FF method.

Foveon X3 Sensors, **With** Prefilter (continued)



thicker curves = Foveon X3 Prefiltered, FF method
thinner = 2 degree observer
long-short = ω1, long dashes = ω2, short dashes = ω3



Thin = 2 deg Observer, orthonormalized
Thicker = 2 deg Observer approximated by Foveon X3 Prefiltered, Fit first, then orthonormalized, then fit again. The second fit should be the same as the first, but for orderly software, the fit is repeated.
long-short = ω1, long dashes = ω2, short dashes = ω3

So, In Summary:

- A general idea was presented at CIC 12, to use orthonormal color matching functions as a path to graphing vectorial sensitivities, vectorial stimuli, and the Locus of Unit Monochromats, all in Jozef Cohen's color space.
- Orthonormalizing the camera's sensitivities leads to an LUM for the camera. If it's the same as human, the Maxwell-Ives criterion is satisfied. Where it deviates, the deviations have meaning for the camera designer.
- Well-known methods give a "figure of merit" for a camera's color fidelity. The LUM method can give a figure of merit and much more.
- Orthonormal functions make many derivations easy. For example the camera's LUM or "best fit" functions can be targets for transformations from camera signals to human color space. It is easy algebra to find the transform matrix from sensors to best fit. See the proceedings article and http://www.jimworthey.com .
- Another important application of vectorial color is color rendering [3]. **If you are designing copiers,** the orthonormal basis of your sensors could be used as a starting point for color rendering analysis. You could easily see how your light source affects color contrast and fidelity. Detailed color rendering examples are on the web site.
- In all these methods, facts are revealed, while hidden assumptions and arbitrariness are absent.
- This poster's on the web site, too.

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